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Research Paper

Analytical solution for one-dimensional vertical electro-osmotic drainage under unsaturated conditions

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ABSTRACT

In this study, exponential functions were incorporated in a one-dimensional model to represent the relationships of hydraulic and electro-osmotic conductivities with matric suction and the soil-water characteristic curve. Analytical solutions for pore water pressure, water content and drainage were derived, and laboratory tests were performed to verify the effectiveness of these solutions. Finally, a parametric study indicated that the soil-water characteristics of the soil had a remarkable impact on the drainage behaviour of electro-osmosis under unsaturated conditions and that the proposed analytical solutions could be used to design electro-osmotic drainage systems in unsaturated soils.

1. Introduction

Electro-osmosis is an electrically induced process in which pore water is driven from the anode to the cathode through the dissolved electrolytes, thus leading to the dewatering and consolidation of saturated soil or other porous media. Casagrande [1] used this phenomenon to consolidate soft soil and enhance its geotechnical properties for engineering purposes. Since then, electro-osmosis has been successfully used for soft ground improvement, slope stabilisation, dewatering of tailings and sludge, pile foundation and soil remediation [2–12]. However, problems such as electrode erosion, low efficiency, uneven consolidation and unsatisfactory treatment results widely exist in practical applications, and improvement methods such as polarity reversal, application of alternating current, injection of a chemical solution and combination with other consolidation methods are highly desired by geotechnical engineers [13]. Recently, electro-osmosis has been used in unsaturated soils to enhance the performance of compacted clay liners at polluted sites by electrokinetic barriers [14–16], improve the stability of partially saturated slopes [17–18], reduce the adhesion of excavated clay materials on steel surfaces of tunnel driving machines [19] and decrease the water content of over-wet subgrade fill or expansive soil [20–23].

On the basis of the assumption that the pore water flow resulting from the hydraulic gradient and the electrical gradient can be superimposed linearly, the governing equation for electro-osmotic consolidation was developed, and many analytical solutions were derived using different conditions to analyse the development of pore water pressure and the degree of consolidation [24–31]. Esrig [24] first proposed a one-dimensional theoretical model and derived the solution for the excess pore water pressure under different boundary conditions. Wan and Mitchell [25] investigated the coupling effect of surcharge preloading and electro-osmotic consolidation. Following their pioneering work, several analytical models were proposed for electro-osmotic consolidation, including the two-dimensional model in a vertical plane [26–27] and the two-dimensional model in a horizontal plane [28]. Considering that prefabricated vertical drain and electric vertical drain are often installed in an equilateral triangular pattern in the ground, axisymmetric models were developed and the corresponding analytical solutions were derived [11,29]. Recently, the non-linear variations in soil compressibility and hydraulic and electro-osmotic conductivities were incorporated into a one-dimensional model, and analytical solutions for excess pore water pressure and degree of consolidation were derived [30,31]. These mathematical analyses generated significant knowledge pertaining to electro-osmotic consolidation and provided useful formulas for engineering design. However, all these analytical solutions were derived based on the assumption that the soil is fully saturated. The flow of pore water from the anode to the cathode changes the soil from a saturated state to an unsaturated state during electro-osmosis, thus leading to a decrease in the degree of saturation and non-linear variations in soil properties such as hydraulic

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conductivity and electro-osmotic conductivity [32–37]. Such variations inevitably affect the development of pore water pressure and drainage during electro-osmosis, and predictions from the existing analytical solutions with the assumption of fully saturated soil would be inaccurate. However, to the best of our knowledge, there is no existing analytical solution for electro-osmotic drainage under unsaturated conditions.

In this study, a one-dimensional model for electro-osmotic drainage in the vertical direction considering unsaturated effects was proposed on the basis of the conservation of fluid mass, Darcy's law and the electro-osmotic flow equation. Then, exponential functions were used to represent the relationships of hydraulic and electro-osmotic conductivities with matric suction and the soil-water characteristic curve, and these were incorporated into the one-dimensional model. The analytical solutions for pore water pressure, water content and drainage were derived, which were reasonably well comparable with the experimental results. Finally, a parametric study was conducted to investigate the influence of the soil-water characteristics on the drainage behaviour of electro-osmosis under unsaturated conditions.

2. Theoretical analysis

Similar to previous studies, a schematic diagram of a one-dimensional model for electro-osmotic drainage was developed in this study as shown in Fig. 1, with the anode on the bottom and the cathode on the top [30,31]. The bottom boundary is impermeable, and the top boundary is permeable. As in previous studies, the z axis was directed downwards [26,31]. Thus, the coordinate of any point in this one-dimensional model can be represented using a positive z value, which is consistent with the depth of the model, H. The following assumptions were made to develop the analytical model for electro-osmotic drainage under unsaturated conditions.

1. The soil is homogeneous, and under unsaturated conditions, the electrical properties of the soil mass are constant over time.
2. The pore water and soil grain are incompressible, the deformation of the soil is neglected and the drainage of pore water occurs in the vertical direction.
3. The velocity of pore water flow due to electro-osmosis is directly proportional to the electrical gradient and can be superimposed linearly due to the hydraulic gradient.
4. The pore water flow caused by the thermal gradient and chemical concentration gradient is neglected.
5. The differential equation describing electro-osmotic flow in unsaturated soil obeys the Richards' equation.
6. The pore air pressure in the soil is kept unchanged, and a constant atmospheric pressure is maintained.

For the configuration shown in Fig. 1, a voltage $V = V_0$ is applied between the electrodes. In this case, the electric field intensity is expressed as follows:

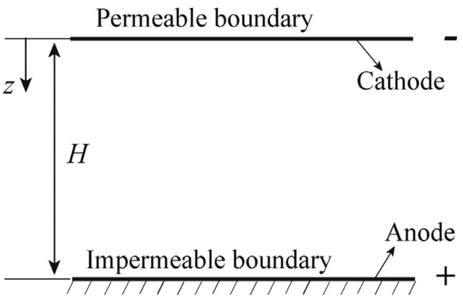

Fig. 1. One-dimensional model for electro-osmotic drainage.

$$\boldsymbol{E} = -\nabla V = -\frac{\partial V}{\partial z}\boldsymbol{i} \tag{1}$$

where $\boldsymbol{E}$ is the intensity of the electric field (in V/m) and $\boldsymbol{i}$ is a unit vector in the z direction. The negative sign in Eq. (1) denotes that the direction of the field intensity vector is from the anode (positive electrode) to the cathode (negative electrode). From assumption (1) that the electrical properties of the soil are homogeneous and constant over time, the electric field can be approximated as uniform within the soil mass. Thus, the distribution of the electrical voltage gradient between electrodes is assumed to be linear during the electrokinetic process. This assumption is in agreement with theory and the experimental observation at lower electric field intensities (typically, $|\boldsymbol{E}| < 100$ V/m) [26]. In addition, considering the voltage loss at the electrode-soil contact, the magnitude of the electric field intensity is usually lower than the total applied voltage divided by the electrode spacing, V_0/H (H is the height of the model), which has been observed in many laboratory and field tests [39].

By using the electric field intensity expressed in Eq. (1) and on the basis of the proportionality between the electrically induced velocity of water flow through the soil and the voltage gradient, the combined pore water flow induced by the hydraulic and electrical gradients during electro-osmosis can be described as follows [24]:

$$v_z = -\frac{k_w}{\gamma_w}\frac{\partial(u - \gamma_w z)}{\partial z} - k_{eo}\frac{\partial V}{\partial z} \tag{2}$$

where v_z is the pore water flow in the vertical direction; γ_w is the unit weight of water; u is the pore water pressure (in such unsaturated situation, the matric suction is equal to the negative pore water pressure); and k_w and k_{eo} are the hydraulic and electro-osmotic conductivities of unsaturated soil, respectively.

On the basis of the conservation of fluid mass and assumptions (3) and (5) and neglecting the deformation of unsaturated soil, the one-dimensional non-linear differential equation that describes water flow in unsaturated soil during electro-osmotic drainage is expressed as follows:

$$\frac{\partial}{\partial z}\left[\frac{k_w}{\gamma_w}\frac{\partial(u - \gamma_w z)}{\partial z} + k_{eo}\frac{\partial V}{\partial z}\right] = \frac{\partial\theta(z,t)}{\partial t} = \frac{\partial\theta}{\partial u}\frac{\partial u}{\partial t} \tag{3}$$

where θ is the volumetric water content, $\partial\theta/\partial u$ is the storativity and t is the time.

Both the hydraulic conductivity and the volumetric water content of unsaturated soil are assumed to be functions of the matric suction [38]

$$k_w(u) = k_s e^{\alpha u} \tag{4a}$$

$$\theta(u) = \theta_r + (\theta_s - \theta_r)e^{\alpha u} \tag{4b}$$

where θ_s and θ_r are the volumetric water content at saturation and the residual volumetric water content, respectively; k_s is the hydraulic conductivity at saturation and α is the desaturation coefficient that represents the pore size distribution.

Similar to hydraulic conductivity under unsaturation conditions, electro-osmotic conductivity can also be defined as the product of the intrinsic conductivity, depending on the microstructural properties of the porous medium, and a non-dimensional relative conductivity coefficient, which accounts for the effects of partial saturation on soil electro-osmotic conduction properties. According to the experimental results obtained in electrokinetic filtration tests performed on unsaturated specimens [32,33], the exponential relationship between the electro-osmotic conductivity and the matric suction is also satisfied.

$$k_{eo}(u) = k_e e^{\beta u} \tag{5}$$

where k_e is the electro-osmotic conductivity at saturation and β is the relative electro-osmotic conductivity exponent. To facilitate the development of analytical solutions, the relative electro-osmotic conductivity exponent β is assumed to be equal to the desaturation coefficient α in