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# Numerical analysis of reinforced concrete beams strengthened in shear by externally bonded (EB) fibre reinforced polymer (FRP) sheets

## *Análisis numérico de vigas de hormigón armado reforzadas a cortante externamente mediante laminados de polímeros reforzados con fibras (PRF)*

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**Abstract**

In this paper, a fibre beam model previously developed by the authors for the nonlinear analysis of strengthened elements, including the effects of shear, is used to predict the response of reinforced concrete (RC) beams strengthened in shear with fibre reinforced polymers (FRP) sheets. This model has been extended not only for wrapped configurations but also for debonding failure in order to allow for its application to beams strengthened with U-shaped and side-bonded configurations. When simulating existing experimental tests, the model reproduces, with reasonable accuracy the behavior of the beams, being then a useful tool for practical engineering.


*Keywords:* Strengthening; Shear; FRP sheets; Debonding; Fibre beam model

**Resumen**

En este artículo se presenta la extensión de un modelo de filamentos y barras para el análisis no lineal de elementos reforzados externamente a cortante con laminados de polímeros reforzados con fibras, teniendo en cuenta los efectos del cortante. Este modelo se ha extendido no solo para configuraciones que envuelven de forma completa la sección con el laminado de polímeros reforzados con fibras, sino también para los casos en que se puede producir desprendimiento prematuro del laminado, para permitir su aplicación en vigas reforzadas en U o con laminados adheridos en las almas. Al simular ensayos experimentales existentes, el modelo reproduce, con una exactitud razonable, el comportamiento de las vigas reforzadas y, por tanto, es una herramienta útil para la ingeniería práctica.


*Palabras clave:* Refuerzo; Cortante; Láminas de PRF; Desprendimiento prematuro; Modelo de filamentos y barras

## 1. Introduction

The shear strengthening by means of FRP sheets or laminates can be performed in different configurations: (a) sheets fully wrapping the cross-section (wrapped); (b) sheets or L-shaped laminates bonded on the lateral sides and the bottom surface of the beam (U-shaped); and (c) sheets or laminates bonded in the lateral sides of the cross-section (side-bonded). The sheets and laminates can be bonded in a continuous or discontinuous manner.

In the case of wrapped configurations, the shear failure is accompanied by FRP rupture or the FRP system can fail due to the rupture of the fibres at the round corner of the section. In

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U-shaped or side-bonded configurations, the FRP may debond before reaching its ultimate capacity.

A fibre beam model developed by the authors for the nonlinear analysis of RC and strengthened elements including the effects of shear [1–4] has been improved to account for strengthening by means of FRP sheets. The model has been validated in [5,6] through the analysis of RC beams strengthened in shear with FRP sheets involving different configurations by means of the modelization or the experimental campaign of Alzate [7], Khalifa and Nanni [8], and Matthys [9]. The model reproduces, with good accuracy, the experimental failure loads, the load-deflection behavior and the strains in stirrups and FRP with increasing load until failure. It also reflects the load-sharing between inner transversal steel reinforcement and EB FRP before and after premature debonding failure. This achievement is important due to its simplicity and computational speed to be applied at true scale structural analysis, making it an attractive tool for practical engineering. This paper briefly explains the model and shows some of the results of their application to simulate some existing experimental results.

## 2. Fibre beam model

### 2.1. Fundamentals of the model

This model is based on a flexural fibre beam model [10] which considers the interaction axial force – shear – bending moment (N-V-M) and uses a displacement-based FE formulation for the nonlinear phased analysis of concrete frame structures. The detailed formulation and validation of the 1D model with shear critical benchmarks was presented elsewhere [1–4]. Only a brief description of the fundamentals of the model is presented here.

Fig. 1 presents the general characteristics of the model for different levels of analysis: element, section, fibre and material. Regarding the element level, the model is based on the Timoshenko beam theory with the cross-section discretized into fibres, the longitudinal reinforcement simulated by means of filaments and transversal reinforcement considered smeared in concrete. At the sectional level, a shear-sensitive model accounts for the nonlinear force interaction (N-V-M). The plane-section theory, that allows determining the longitudinal strains at each fibre as a function of the generalized strains of the section, is coupled with a constant shear stress constraint along the cross-section. Filaments of longitudinal reinforcement are only submitted to axial strains and stresses, following the plane section theory. Transverse reinforcement (internal steel stirrups and/or EB FRP) is accounted through its volumetric ratio $\rho_{st}$ and is submitted to axial stresses $\sigma_{zst}$. Compatibility requirements impose that the vertical strain $\varepsilon_z$ in concrete is equal to the strain in the transverse reinforcement. The computed shear stresses $\tau_{xz}$ must equate the imposed shear stresses given by the fixed stress constraint $\tau^*$ of the sectional hypothesis. By guaranteeing these two requirements, the vertical axial strain $\varepsilon_z$ and shear strain $\gamma_{xz}$ of each fibre are outputted. This determination is not linear and an iterative procedure within the fibre level is needed.

Pertaining to the material simulation, a smeared and rotating crack approach is considered for concrete. Then, the cracked concrete is simulated as a continuous material with orthotropic nonlinear uniaxial equivalent characteristics considering full rotation of cracks. The Hognestad parabola is considered for concrete in compression. Lateral effects of softening [11] and strength enhancement [12] factors are included. When FRP strengthening is placed by means of a wrapped configuration, the increment of both peak strength and ultimate strain of concrete due to the confinement action is considered through the model of Spoelstra and Monti [13]. A linear response is assumed for uncracked concrete in tension and a tension stiffening curve [14] is considered for the remaining stresses in the cracked stage. Longitudinal and transverse reinforcements (steel and FRP) are under 1D stress–strain states determined through linear uniaxial constitutive equations, with kinematic hardening for steel. Perfect bond is assumed between the concrete and the steel reinforcement.

### 2.2. Debonding failure

As experimentally observed in existing experimental programmes, U-shaped and side-bonded configurations of FRP usually fail due to debonding after the formation of a critical shear crack. For shear strengthening, the debonding failure initiates once the shear critical crack opens. Then, the laminate debonds if the FRP bonded length from the shear crack to the laminate end is not enough to anchor or transfer the tensile force acting on the FRP. In the side-bonded case, debonding can be observed at both sides of the critical shear crack. In the U-shaped case, debonding occurs in the upper side of the shear crack.

The debonding failure approach implemented in the present model is that proposed by Oller et al. [15]. This model was originally developed to predict debonding failure at the laminate end in beams externally strengthened by FRP in bending. This formulation can also be applied when predicting debonding for FRP shear strengthening. For U-shaped configurations, the bonded length $L_b$ of each strip is the bonded length above the critical shear crack. For side-bonded configurations, the bonded length of each strip is the minimum length of the laminate above or below the critical shear crack.

The maximum transferred force $F_{max}$ along the bonded length $L_b$, can be expressed as [15]:

$$F_{max,Lb} = b_f\sqrt{2G_f E_f t_f}\begin{cases} sin\left(\dfrac{\pi}{2}\dfrac{L_b}{L_{lim}}\right) & L_b \leq L_{lim} \\ 1 & L_b > L_{lim} \end{cases} \quad (1)$$

$$L_{lim} = \frac{\pi}{2}\frac{\sqrt{2G_f E_f t_f}}{\tau_{LM}} \quad (2)$$

where $b_f$ is the FRP width; $t_f$ is the FRP thickness; $E_f$ is the FRP modulus of elasticity; $\tau_{LM}$ is the maximum shear stress at the interface given by Eq. (3); $G_f$ is the fracture energy or energy by unit area necessary to separate the laminate from the support given by Eq. (4). Units are in N and mm.

$$\tau_{LM} = C_{\tau_{LM}}\left(\frac{1}{f_{ctm}} + \frac{1}{f_{cm}}\right)^{-1} \quad (3)$$