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A flexible modelling approach for capturing plastic anisotropy and strength differential effects exhibited by commercially pure titanium

Niko Manopulo*, Christian Raemy, Pavel Hora

Institute of Virtual Manufacturing, Tannenstrasse 3, ETH Zurich, 8092 Zurich, Switzerland

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ABSTRACT

Commercially pure titanium exhibits strong anisotropy, strength differential as well as texture evolution, which make the development of predictive simulation models a challenging task. A new modeling approach is proposed based on the homogeneous distortion of well-established yield surface descriptions, in order to achieve a flexible tool to capture all the mentioned effects. The models are calibrated based on uniaxial tensile and compression tests and are validated based on the earing profile and thickness distribution of cup drawing experiments.

1. Introduction

Commercially pure titanium and more generally hcp-metals, have excellent strength-to-weight ratio, as well as corrosion resistance properties. These materials are therefore the preferred choice in medical and aerospace industries. The numerical simulation of forming processes, which is nowadays standard practice for materials like steel and aluminum, is hardly applied as far as hcp-metals are concerned. This is primarily due to the fact that these materials exhibit strong anisotropy, strength differential (SD), as well as non-proportional hardening effects which cannot be captured by the classical yield surface models such as the Hill family (e.g. Hill (1948); 1979)), the Barlat family (e.g. Barlat and Lian (1989); Barlat et al. (1991); 2003); Banabic et al. (2005); Barlat et al. (2007)) or others (e.g. Vegter and van den Boogaard (2006)).

The SD effect, is widely attributed to the loading path dependency of deformation twinning as suggested by Hosford and Allen (1970). Especially in titanium, deformation induced twinning is exclusively observed under compressive loads, whereas tensile deformation is primarily governed by slip. Phenomenological yield surface models accounting for this effect have been introduced by Cazacu and Barlat (2004), who modified Drucker's model to allow yield assymetry. A later modification of the model enables capturing yield anisotropy as well (Cazacu et al., 2006). Although this model features significantly more freedom than classical models, it still lacks flexibility to model the real behavior of titanium alloys. In a more recent publication the authors augmented the set of linear transformations applied to the stress deviator, achieving a better agreement with experimental results (Plunkett et al., 2008). Nixon et al. (2010), on the other hand, improved the model proposed by Cazacu and Barlat (2004) introducing a linear transformation of the stress deviator in the formulation. An alternative approach has been proposed by Bai and Wierzbicki (2008), who described the SD effect as a function of the stress triaxiality and Lode parameter. More recently Khan et al. (2012) proposed a model combining an isotropic but asymmetric function with a symmetric but anisotropic one in order to appropriately capture both effects. Yoon et al. (2014) also investigated the problem and proposed an approach involving both linear transformations and dependency on the first and third stress invariants.

As delineated in the previous paragraph, many different approaches have been proposed in the past for modelling strength differential and anisotropy in hcp-metals. Most of these approaches consider the invariants of linearly transformed stress deviator tensors and as such overlap to a certain degree. The aim of this contribution is to provide a clear separation between the modeling of SD effect and anisotropy. To achieve this a modified version of the HAH model introduced by Barlat et al. (2011) is proposed as a flexible framework to construct asymmetric yield surfaces, based on well-established anisotropic yield models. The models are calibrated using uniaxial tension and compression tests and are compared against alternative approaches based on cup drawing experiments.

2. Material properties

The material properties discussed by Raemy et al. (2017) have been used in this work to calibrate and validate the proposed models. A 3mm thickness sheet of a TiCP-Grade4 has been used for the

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* Corresponding author.
E-mail address: manopulo@ivp.mavt.ethz.ch (N. Manopulo).

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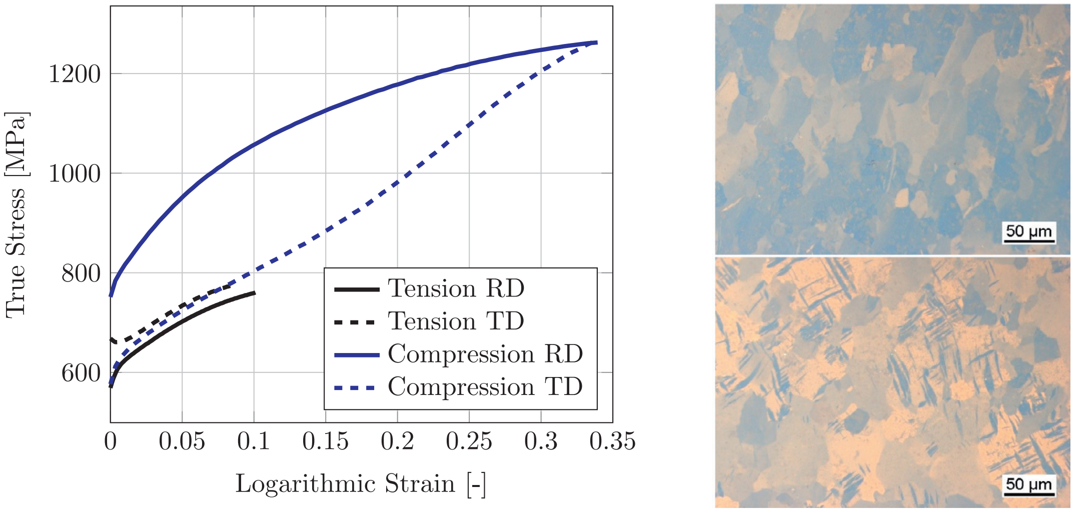

(a) Yield curves in tension and compression (b) RD (above) and TD (below)

Fig. 1. Flow curves and optical micrographs at 0.15 compression strain of TiCP-Grade4 (Raemy et al., 2017).

Table 1
Yield strengths and Lankford parameters of Ti-CP Grade 4 in different directions (*assumed).

Sense	σ_0	σ_{45}	σ_{90}	σ_b	R_0	R_{45}	R_{90}
Tens.	569 MPa	620 MPa	651 MPa	800 MPa	1.05	2.10	2.25
Compr.	751 MPa	600 MPa	580 MPa	606 MPa*	1.05*	2.10*	2.25*

tensile tests, carried out with standard dog-bone specimens wire-cut out of the blank at each 45° from rolling direction (RD). Additionally uniaxial compression tests have been conducted using a dilatometer equipment using prismatic specimens of size of 3 × 3 × 5 mm. A separate sheet of 5 mm thickness[1] has been used for the through-thickness compression tests using the same specimen geometry. Fig. 1a depicts the measured flow curves in tension and compression, in rolling (RD) as well as transverse (TD) directions. It is noted that there is pronounced initial yield asymmetry in RD, whereas in TD the initial yield stresses are very close. Furthermore it is seen that the compression test in transverse direction exhibits accelerated hardening due to deformation twinning, delivering a typical s-shaped curve. This is attributed to the twinning structures which locally hinder dislocation movement (Salem et al., 2006). Optical micrographs executed on specimens in rolling and transverse direction at approx. 0.15 strain also confirm the presence of the latter (see Fig. 1b).

All the data to be used in the identification of the constitutive models has been summarized in Table 1. Please note that the starred values have not been measured directly but assumed. Namely the R-values in compression have been set equal to the values in tension and the equibiaxial stress in compression has been scaled from the corresponding value in tension, assuming that the T/C ratio is the same as in uniaxial RD.

As larger strain values could be reached in this test, the uniaxial compression curve in RD has been used as reference to identify the isotropic hardening model according to Hockett and Sherby (1975), which reads

$$\sigma_y = B - (B - A)e^{-m \cdot \bar{\varepsilon}^p} \tag{1}$$

The corresponding parameters can be found in Table 2. Distortional hardening effects, which are clearly present during the deformation of TiCP-Grade4 have been left out of the scope of this work. For the particular process at hand anisotropic hardening does not greatly influence the results. The interested reader is referred to Raemy et al. (2017) for a more detailed discussion of this phenomenon.

Table 2
Hockett–Sherby parameters of flow curve for uni-axial compression in rolling direction (Ti-CP Grade 4).

A	B	m	p
751 MPa	1364 MPa	4.34	0.801

3. Modeling approach

The proposed yield surface model as well as its numerical implementation for explicit FEM applications with brick elements is discussed in this section. For reasons of brevity details about the base yield surface descriptions are left out of scope. The reader is advised to consult the corresponding literature for a more thorough review.

3.1. Proposed yield surface model

In their recently published work Barlat and co-workers proposed the HAH model to capture distortional hardening due to Bauschinger and latent hardening effects (Barlat et al., 2011; Lee et al., 2012; Barlat et al., 2013; Manopulo et al., 2015). The model reads

$$\bar{\sigma}^q(\boldsymbol{s}) = \phi^q(\boldsymbol{s}) + g_1^q \left| \boldsymbol{h} : \boldsymbol{s} - \left| \boldsymbol{h} : \boldsymbol{s} \right| \right|^q + g_2^q \left| \boldsymbol{h} : \boldsymbol{s} + \left| \boldsymbol{h} : \boldsymbol{s} \right| \right|^q, \tag{2}$$

where $\boldsymbol{s}$ is the stress deviator, ϕ is any homogeneous yield function, g_1, g_2 and $\boldsymbol{h}$ are dynamic state variables, which are subject to

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[1] Since the yield stress in rolling direction for both sheets varied within few percents they were assumed to have comparable properties.

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