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Advanced equivalent source methodologies for near-field acoustic holography

Nicolas P. Valdivia

Code 7130, Naval Research Laboratory, Washington, DC 20375, USA

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ABSTRACT

The equivalent source method (ESM) based near-field acoustic holography (NAH) has been used successfully in the reconstruction of the acoustical field over arbitrarily shaped vibrating structures from measurements of the pressure field on a nearby conformal surface. In this work we study the use of the ESM dipole and a combined source representation for NAH reconstructions. Using numerically generated data from an elastic spherical shell acoustic response, the studies are focused on the correct positioning and distribution of the source points in relation to the vibrating surface, accuracy over problematic frequencies and spatial aliasing. Finally, we utilize physical data from a vibrating ship-hull structure to validate the proposed ESM reconstructions.

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1. Introduction

The reconstruction of the acoustic field (pressure, normal velocity, and intensity) plays an important role in the early stage of effective noise control. The well known technique of Nearfield Acoustical Holography (NAH) [1] is a crucial tool in the understanding of the basic mechanisms of acoustic radiation at a vibrating surface. For NAH, the pressure field radiated or scattered from an object is measured on an imaginary surface outside (for exterior problems) or inside (for interior problems) the source. This pressure hologram is used to reconstruct the acoustic field on the body of the source by solving an ill-posed problem. Originally, NAH was formulated over separable geometries (like planes, cylinders or spheres), where the solution relies on the expansion of the pressure field in terms of a complete set of eigenfunctions along with a knowledge of the analytical form of the Neumann or Dirichlet Green's function (see Ref. [2]). The reconstructions are very efficient, requiring only fractions of a second of computation time per frequency. Sources with boundaries which vary appreciably in shape from one of these separable geometries can be attacked using boundary element methods (BEM) (see Refs. [3,4]) which yields a numerical form of the appropriate Green's transfer function, along with the singular value decomposition (SVD) which provides orthonormal eigenfunctions for the geometry. With these eigenfunctions, the methodology of the inversion is identical to that used for the separable geometry. One pays, however, for the expanded versatility by a considerable increase in computation time.

As argued previously in Ref. [5], for the classical BEM implementation of NAH it is a common practice to compute the discretization of the Green's transfer function. In addition to the BEM discretization of the surface integrals, the construction of the transfer functions requires an explicit inversion of a matrix system that can be a prohibitive practice for large reconstructions. There are some alternative representations of the classical BEM integral representation known as the implicit-indirect formulations in which fictitious density distributions (single and/or double layer integral representations) are used as an intermediate step between the measured pressure and the reconstructed velocity or pressure fields (see Refs. [5–9]). Although these formu-

E-mail address: nicolas.valdivia@nrl.navy.mil.

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lations only require the BEM discretization of the required integral equations there are still many issues related to the effective evaluation of hyper-singular kernels that result in some of the surface integrals. Finally, there exist a "discrete" methodology known as the equivalent sources method (ESM) which produces matrix relations similar to the BEM implicit-indirect formulations. In conventional ESM the acoustic field is represented by a set of monopole sources distributed over a surface close to the radiating structure surface. The original theoretical justifications of ESM can be found in the works of Vekua [10], Kupradze [11,12], and Müller [13], which are based on the classical theory of analytical continuation (see Ref. [14]). A different justification was presented in our previous work [15] where the monopole source representation is understood as an approximation to the single layer integral equation.

The theory developed by Kupradze [12] was formulated for monopoles (which are more common in practice), but can be applied to dipoles or a combination of sources. Indeed, we can find results in Ref. [16] or [14, Chapter 2] where the use of combination of sources were proposed to avoid the errors related to problematic frequencies for the forward scattering problems. Alternatively these errors can be avoided by modifying the shape of the source surface or adding interior sources [17]. The goal of this work is to validate the dipole and combined ESM representations for NAH reconstructions. The later will be a more robust approach to treat multiple frequencies reconstructions where we don't require to modify the source surface. First, in section 3, we justify using integral estimates that the dipole and combined source ESM representation are constant element approximations to the known implicit-indirect BEM integral representations. Then, in section 5, we perform multi-frequency reconstructions for numerically generated data to justify the accuracy of the proposed ESM representation for noiseless and gaussian-noise data. Also in this section we study the frequency limits where spatial aliasing error dominates the ESM reconstructions. Finally, in section 6, we validate further the proposed reconstruction methods using the resultant radiated data from a model ship-hull excited by a shaker.

2. Boundary integral representations

Let G be a domain in $\mathbb{R}^3$, interior to the boundary surface Γ where we assume that Γ is allowed to have edges and corners. We denote as $G^+ := \mathbb{R}^3 \backslash G$ the unbounded exterior medium outside of G. The behavior of a time-harmonic acoustic wave p with $e^{-i\omega t}$ dependence, where ω is the angular frequency, is described by the Helmholtz equation

$$\nabla^2 p + k^2 p = 0, \quad \text{in } G \left(\text{or } G^+\right) \tag{1}$$

where $k = \omega / c$ and c is the speed of sound in the medium. The radiating nature of the wave p in G^+ is modeled by Eq. (1) and the Sommerfeld radiation condition [18,19].

2.1. Direct formulation

The radiated wave in Eq. (1) can be represented using the Helmholtz-Kirchhoff Integral Equation (HIE) (as originally described by Kupradze [20–22]), for the exterior problem

$$\left(K_\Gamma p\right)(\mathbf{x}) - i\omega\rho\left(S_\Gamma v\right)(\mathbf{x}) = \begin{cases} p(\mathbf{x}), & \mathbf{x} \in G^+, \\ (1 - \Omega(\mathbf{x}))\, p(\mathbf{x}), & \mathbf{x} \in \Gamma, \\ 0, & \mathbf{x} \in G, \end{cases} \tag{2}$$

and for the interior solution

$$i\omega\rho\left(S_\Gamma v\right)(\mathbf{x}) - \left(K_\Gamma p\right)(\mathbf{x}) = \begin{cases} 0, & \mathbf{x} \in G^+, \\ (1 - \Omega(\mathbf{x}))\, p(\mathbf{x}), & \mathbf{x} \in \Gamma, \\ p(\mathbf{x}), & \mathbf{x} \in G, \end{cases} \tag{3}$$

where v is the normal velocity and the operator notation S_Γ and K_Γ is used as in Ref. [18]

$$\begin{aligned} \left(S_\Gamma \phi\right)(\mathbf{x}) &:= \int_\Gamma \Phi(\mathbf{x}, \mathbf{y})\phi(\mathbf{y})\mathrm{d}S(\mathbf{y}), \\ \left(K_\Gamma \phi\right)(\mathbf{x}) &:= \int_\Gamma \frac{\partial \Phi(\mathbf{x}, \mathbf{y})}{\partial \mathbf{n}(\mathbf{y})}\phi(\mathbf{y})\mathrm{d}S(\mathbf{y}), \end{aligned} \tag{4}$$

and

$$\Phi(\mathbf{x}, \mathbf{y}) = \frac{\exp\left(ik|\mathbf{x} - \mathbf{y}|\right)}{4\pi\, |\mathbf{x} - \mathbf{y}|}. \tag{5}$$

In Eqs. (2) and (3), ρ is the mean fluid density and in Eq. (3) $\mathbf{n}$ is the unit normal with direction shown in Fig. 1. Finally in Eqs. (1) and (2), $\Omega(\mathbf{x})$ is the solid angle coefficient given by the integral formula

$$\Omega(\mathbf{x}) = -\int_\Gamma \frac{\partial}{\partial \mathbf{n}(\mathbf{y})}\left(\frac{1}{4\pi|\mathbf{x} - \mathbf{y}|}\right)\mathrm{d}S(\mathbf{y}), \quad \mathbf{x} \in \Gamma. \tag{6}$$

$\Omega(\mathbf{x})$ will be $1/2$ for all $\mathbf{x} \in \Gamma$, when Γ is a smooth surface.