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On the extension of cylindrical acoustic waves to acoustic-vortical-entropy waves in a flow with rigid body swirl

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A B S T R A C T

The noise of jet and rocket engines involves the coupling of sound to swirling flows and to heat exchanges leading in the more complex cases of triple interactions to acoustic-vortical-entropy (AVE) waves. The present paper presents the derivation of the AVE equation for axisymmetric linear non-dissipative, compressible perturbations of a non-homentropic, swirling mean flow, with constant axial velocity and constant angular velocity for a perfect gas with constant density. The axisymmetric AVE wave equation is obtained for the radial velocity perturbation, specifying its radial dependence for any frequency and axial wavenumber. The AVE wave equation in the case of zero axial wavenumber, corresponding to cylindrical AVE waves, has no singularities for finite radius, including the sonic radius, where the isothermal Mach number for the swirl velocity is unity. The exact solution of the AVE wave equation for the fundamental axisymmetric mode with zero axial wavenumber is obtained without any restriction on frequency, as series expansions of Gaussian hypergeometric type: (i) covering the whole flow region; (ii) specifying the wave field at the sonic radius; (iii) specifying near-axis and asymptotic scaling for small and large radius. Using polarization relations among wave variables specifies exactly and allows the plotting of the perturbations of: (i,ii) the radial and azimuthal velocity; (iii,iv) pressure and mass density; (v,vi) entropy and temperature. Thus the extension of cylindrical acoustic waves, that are specified by Bessel functions, to cylindrical acoustic-vortical-entropy waves, is specified by Gaussian hypergeometric functions.

1. Introduction

The noise of aircraft engines is a major limitation on airport operations, and the subject of ever more stringent certification rules, aiming to limit the total noise exposure as air traffic grows. The noise of the rocket engines of space launchers is sufficiently high to cause structural damage and require payloads like satellites to be tested for acoustic fatigue in reverberant chambers. The literature on aircraft and rocket engine noise usually considers purely acoustic waves, although coupling with other modes occur in: (i) inlet ducts due to the shear flow in the wall boundary layers; (ii) in turbine exhausts due to the downstream swirling flow; (iii) in the combustion chambers and other heat generation and exchange processes involving non-isentropic flows. The simplest mean flow for which there are interacting acoustic, vortical and entropy perturbations is an axisymmetric non-homentropic flow with uniform axial velocity and rigid body swirl; this sample problem is of interest in itself relating to

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waves in nozzles with swirl and heat exchange.

There are [1–3] three types of waves in a fluid in the absence of external restoring forces [4,5], namely: (i) sound waves that are longitudinal and compressive; (ii) vortical waves that are transversal, hence incompressible; (iii) entropy modes associated with heat exchanges, hence non-homentropic flow. The acoustic modes receive most attention because for an homogeneous uniform mean flow: firstly, the acoustic modes satisfy the convected wave equation for uniform motion and the classical wave equation in a medium at rest [6–13]; secondly, by the Kelvin circulation theorem the circulation along a loop convected with the mean flow is constant [14–18]; thirdly, in homentropic conditions there are no entropy modes. The most general conditions for the existence of purely acoustic modes, decoupled from vortical-entropy modes, is a potential homentropic mean flow, that may be compressible, and leads to the high-speed wave equation [19–21] that reduces to the convected wave equation [22–24] in two cases: (i) uniform flow; (ii) low Mach number steady non-uniform flow. The presence of vorticity leads to acoustic-vortical waves [25–30], in a compressible sheared [31–44] or swirling [45–57] mean flow. The present paper considers a further extension to acoustic-vortical-entropy waves [57] that specify the stability of a compressible, vortical and non-homentropic mean flow.

The acoustic, vortical and entropy modes [1–3] are decoupled in a medium at rest and become coupled in sheared and/or swirling non-homentropic mean flows. The first derivation of the acoustic-vortical (AV) wave equation [46] used a decomposition of the velocity perturbation into an irrotational and a vortical part. The AV waves were considered for specific swirl profiles [47–49] such as rigid body and potential vortex. These two profiles were also considered as particular cases of AV waves in an axisymmetric mean flow with shear and swirl arbitrary functions of the radius, assuming constant density [49]; the latter condition can be replaced by that of an homentropic mean flow [30]. Besides the propagation [30,45–52], also the generation [53–56] of AV waves has been considered. The more general case of AVE waves has been considered [57] as isentropic non-axisymmetric longitudinally propagating perturbations of a non-homentropic mean flow with shear and swirl arbitrary functions of the radius in the WKB approximation of high-frequency, which excludes critical layers. The present paper derives the AVE wave equation also for isentropic perturbations of a non-homentropic mean flow, restricting to a uniform axial flow and rigid body swirl and axisymmetric modes, without restriction of frequency, showing that a critical layer exists for isothermal sound speed equal to the phase speed based on the Doppler shifted frequency. The critical layer does not exist for zero axial wavenumber, that is for cylindrical AVE waves, in which case the exact solution valid for all frequencies and distances is obtained in terms of Gaussian hypergeometric functions.

The linearized Euler equations (LEE) contain all these modes, but consist of one vector (momentum) and three scalar (continuity, energy and state) equations with six variables (velocity vector, pressure, density and entropy). In this formulation, the 'wave operator' is a 6×6 matrix that cannot be readily compared to a scalar wave equation for one variable like the pressure perturbation. This paper presents a scalar wave equation for a single wave variable (the radial velocity) that generalizes the classic wave equation for sound and the acoustic-vortical wave equation in a swirling flow. This derivation involves elimination among the 6 LEE equations for one variable only, namely the radial velocity, that determines through polarization relations all other variables, namely the perturbations of the density, pressure, temperature, entropy and axial and azimuthal components of the velocity. There is substantial evidence in the literature of the presence of non-acoustic perturbations in nozzle flows, and the derivation of an acoustic-vortical-entropy (AVE) equation aims to address this limitation of current wave equations, by allowing the interaction of all three effects.

The radial dependence of the fundamental axisymmetric mode of non-dissipative AVE waves, in a uniform mean flow with rigid body swirl of a perfect gas with constant density is specified exactly, without any restriction on frequency, by a linear second-order differential equation with four regular singularities, of which one is a critical layer. The critical layer does not exist for zero axial wavenumber, corresponding to cylindrical AVE waves that depend only on time and radial distance. The radial dependence of the cylindrical AVE equation is thus specified by a linear second-order differential equation with three regular singularities, whose solution is (Section 3) specified by Gaussian hypergeometric functions [58–60]. These replace the Bessel functions [61–63] that specify acoustic cylindrical waves in an homentropic medium at rest or in uniform motion but without swirl. The three singularities of the cylindrical AVE wav equation are firstly the origin specifying the inner wave field, secondly the point at infinity specifying the asymptotic wave field, and thirdly the sonic point where the swirl velocity equals the isothermal sound speed. The latter is an apparent singularity of the differential equation [64–66] since the wave field is finite at the sonic condition. The solution around the sonic condition matches continuously the inner and asymptotic wave fields.

Since both the cylindrical AVE wave equation and its solutions are exact, they specify the wave field for all frequencies and radial distances. The cylindrical AVE wave equation is an exact linearization of the equations of motion, and thus its solutions specify the stability of the non-homentropic, non-dissipative uniform flow with rigid body swirl of a perfect gas with constant density. Since the dependence on time is sinusoidal with fixed frequency, the stability for cylindrical perturbations can only be considered in the radial direction. It is shown (Fig. 1) that as the radial distance tends to infinity the perturbations are waves with finite amplitude if the frequency exceeds the angular velocity (with a factor of order unity); conversely, if the frequency is less than the angular velocity, the perturbations grow radially and are unbounded. These conclusions broadly agree with earlier results [67–69] on the stability of vortical flows. The distinction between cylindrical AVE waves with finite radial amplitude and radially unbounded perturbations of the mean flow is illustrated by plotting the radial dependence of the first six modes in a rigid cylindrical duct (Section 4) for the perturbations of the radial and azimuthal velocities, pressure, density, temperature and entropy (Figs. 2–7).