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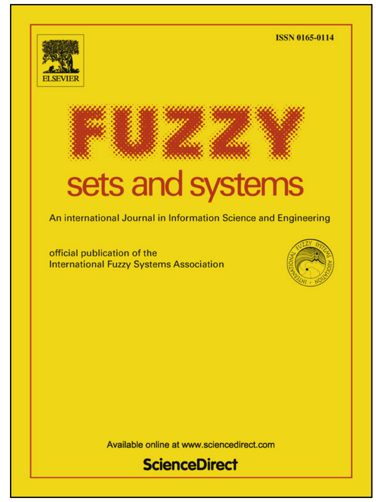

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Type-Reduced Set Structure and the Truncated Type-2 Fuzzy Set

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Abstract

In this paper, the Type-Reduced Set (TRS) of the continuous type-2 fuzzy set is considered as an object in its own right. The structures of the TRSs of both the interval and generalised forms of the type-2 fuzzy set are investigated. In each case the respective TRS structure is approached by first examining the TRS of the discretised set. The TRS of a continuous interval type-2 fuzzy set is demonstrated to be a continuous horizontal straight line, and that of a generalised type-2 fuzzy set, a continuous, convex curve. This analysis leads on to the concept of truncation, and the definition of the *truncation grade*. The *truncated type-2 fuzzy set* is then defined, whose TRS (and hence defuzzified value) is identical to that of the non-truncated type-2 fuzzy set. This result is termed the *Type-2 Truncation Theorem*, an immediate corollary of which is the *Type-2 Equivalence Theorem* which states that the defuzzified values of type-2 fuzzy sets that are equivalent under truncation are equal. Experimental corroboration of the equivalence of the non-truncated and truncated generalised type-2 fuzzy set is provided. The implications of these theorems for uncertainty quantification are explored. The theorem's repercussions for type-2 defuzzification employing the α-Planes Representation are examined; it is shown that the known inaccuracies of the α-Planes Method are deeply entrenched.

Keywords: Type-2 Fuzzy Set, Type-Reduced Set, Type-2 Truncation Theorem, Type-2 Equivalence Theorem, Type-2 Defuzzification, Fuzzy Set Theory

1. Introduction

Type-2 fuzzy sets are an extension of type-1 fuzzy sets in which the sets' membership grades are themselves type-1 fuzzy sets. The concept dates back to Zadeh's seminal paper of 1975 [39]. They take two forms, the interval, for which all secondary membership grades are 1[1], and the generalised, where the secondary membership grade may take any value between 0 and 1. For the computationally simpler interval type-2 Fuzzy Inferencing System (FIS) [31] applications in areas such as control, simulation and optimisation have been developed [1, 4–8, 25]. So far, generalised type-2 fuzzy applications are few in number [22, 26, 31]. This is attributable to the enormous computational complexity of generalised type-2 fuzzy inferencing. Strategies have been developed that reduce the computational complexity of all stages of the generalised type-2 FIS [15, 21, 27, 42]. We believe that the research presented in this paper will lead to further complexity reducing techniques, in turn leading to an increasing number of generalised type-2 FIS applications.

In a Mamdani Type-2 FIS (Figure 1), a crisp numerical input passes through three stages of processing: fuzzification, inferencing, and lastly, the crucial stage of defuzzification. Through defuzzification, the type-2 *aggregated set* produced during the inferencing stage is converted into a crisp

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[1]Interval type-2 fuzzy sets may be regarded as generalisations of interval-valued fuzzy sets [2].