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Transitive blocks and their applications in fuzzy interconnection networks

Sunil Mathew [a,*], Anjali N [a], J.N. Mordeson [b]

[a] *Department of Mathematics, National Institute of Technology Calicut, 673601, India*
[b] *Department of Mathematics, Creighton University, 68178, USA*

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Abstract

In any network, the problem of reduction of flow is more important and frequent than the disconnection of the network. Problems related with maximum bandwidth paths and maximal flows are also very significant. In this article, the authors discuss connectivity concepts related with paths and flows in interconnection networks in a fuzzy framework. Some of the properties and applications of a stable structure called transitive block are discussed. Blocks in fuzzy graphs generalize the concept of blocks in graphs. The existence of a strongest strong cycle in any fuzzy block having at least three vertices is established. Four characterizations previously proposed for a class of blocks in fuzzy graphs are found to be true for a larger class. Two subclasses of θ-fuzzy graphs, namely, connectivity-transitive and cyclically-transitive fuzzy graphs are introduced to get a better understanding of blocks in fuzzy graphs. An application of transitive blocks in interconnection networks is also proposed.

Keywords: Fuzzy graph; Strong cycle; Connectedness; Block; Network

1. Introduction and preliminaries

The problem of finding maximum flow and maximum flow paths between pairs of nodes in a network is widely discussed in the literature. This problem is known as maximum bottleneck path problem [14], maximum bandwidth problem, widest path problem, etc. in different disciplines. Widest paths are necessary in large interconnection networks for providing customized applications. For example, in internet applications like video on demand, large file transfer, availing extra channels, etc. Paths with maximum steady flow should be guaranteed to the paid customer. Problems like maximization of flow through different internally disjoint paths [8], maintaining flow even in partial network damage etc. are also time demanding. In this article, we discuss undirected fuzzy networks of special types. We provide models where the flow is maintained even in the failure of some of the nodes and links in the network. In [17], θ-fuzzy graphs and θ-grids were introduced. This is a continuation of that work. We present a more general-

* Corresponding author.
E-mail addresses: sm@nitc.ac.in (S. Mathew), anjalin0.0@gmail.com (N. Anjali), mordes@creighton.edu (J.N. Mordeson).

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ized structure termed as transitive fuzzy graphs in this paper. Several connectivity properties of this structure are also provided.

Even though problems related with min-max flow algorithms [5], convergence, etc. are studied widely in the computer domain, there is not much mathematical theory available. These algorithms are widely used in internet routing, [1] digital compositing, metabolic path analysis, etc. A graph theoretical analysis of undirected networks can provide more accuracy and robustness to these applications. Also, fuzzy graph models are better to deal with problems of reduction of flows in networks. Classical graph models deal with disconnection rather than reduction of flows.

To achieve a minimal required quality of service, the bandwidths of networks are to be assigned properly. The same topology with different assignments behave differently. A variety of structures have been studied in fuzzy graph theory with variations in the weights assigned to the links. Fuzzy tree, complete fuzzy graph, block, etc. are examples. In this article we propose a transitive block model for bandwidth assignment to any network topology. This assignment keeps a network away from some of the problems arising of bottlenecks and overloading of nodes. An illustration is provided in section 4.

Most of the basic definitions in fuzzy graph theory can be seen in [9–12,16,23]. But for better understanding of this work, we provide some of the basic definitions and results below.

A *fuzzy graph (f-graph)* [7] is a pair $G : (\sigma, \mu)$ where σ is a fuzzy subset of a set S and μ is a fuzzy relation on σ. We assume that S is finite and nonempty, μ is reflexive and symmetric. In all the examples σ is chosen suitably. Also, we denote the underlying graph by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \epsilon S : \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \epsilon S \times S : \mu(u, v) > 0\}$. A fuzzy graph $H : (\tau, \nu)$ is called a partial fuzzy subgraph of $G : (\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for every u and $\nu(uv) \leq \mu(uv)$ for every pair of vertices u and v. We call $H : (\tau, \nu)$ a *fuzzy subgraph* of $G^* : (\sigma^*, \mu^*)$ if $\tau(u) = \sigma(u)$ for every $u \epsilon \tau^*$ and $\nu(u, v) = \mu(u, v)$ for every $(u, v) \epsilon \nu^*$. A *path* P of length n is a sequence of distinct vertices $u_0, u_1, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, \ldots, n$ and the degree of membership of a weakest edge in P is defined as its *strength*, denoted by $s(P)$. If $u_0 = u_n$ and $n \geq 3$, then P is called a *cycle* and a cycle P is called a *fuzzy cycle (f-cycle)* if it contains more than one weakest edge [12]. The *strength of connectedness* between two vertices x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$. An x–y path P is called a *strongest* x–y path if its strength equals $CONN_G(x, y)$ [7]. An f-graph $G : (\sigma, \mu)$ is *connected* if for every x, y in $\sigma^*, CONN_G(x, y) > 0$. Through out, we assume that G is connected. An edge of an f-graph is called *strong* if its weight is at least as great as the connectedness of its end vertices when it is deleted and an x–y path P is called a *strong path* if P contains only strong edges [4].

An edge is called a *fuzzy bridge* of G if its removal reduces the strength of connectedness between some pair of vertices in G. Similarly a *fuzzy cut vertex* w is a vertex in G whose removal from G reduces the strength of connectedness between some pair of vertices other than w [7]. A fuzzy graph having no fuzzy cutvertices is called a block [7]. Sometimes we refer a fuzzy graph which is a block as a fuzzy block. Blocks of a fuzzy graph G are the maximal induced connected subgraphs of G, which are fuzzy blocks [2]. A set of strong edges E is said to be a *fuzzy arc cut (FAC for short)* if either $CONN_{G-E}(x, y) < CONN_G(x, y)$ for some pair of vertices x and y with at least one of them different from the end vertices of edges in E or if $G - E$ is disconnected. A *FAC* with n edges is called an $n - FAC$. Also, a $1 - FAC$ is said to be a *fuzzy bond (f-bond for short)* [20]. It is clear that every fuzzy bond is a fuzzy bridge, but not conversely. For example, in Fig. 1, edge uw is a fuzzy bridge, but not a fuzzy bond. A connected f-graph $G : (\sigma, \mu)$ is a fuzzy tree (f-tree) if it has a fuzzy spanning subgraph $F : (\sigma, \nu)$, which is a tree, where for all edges (x, y) not in F there exists a path from x to y in F whose strength is more than $\mu(x, y)$ [13]. Note that F is the unique maximum spanning tree (MST) of G [21]. A *complete fuzzy graph* (CFG) is an f-graph $G : (\sigma, \mu)$ such that $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ for all x and y [3]. The θ – evaluation of u and v is defined as $\theta(u, v) = \{\alpha : \alpha \in (0, 1]\}$, where α is the strength of a strong cycle passing through both u and v [17]. $Max\{\alpha : \alpha \in \theta(u, v); u, v \in \sigma^*\}$ is defined as the *cycle connectivity* between u and v in $G : (\sigma, \mu)$ and is denoted by $C^G_{u,v}$ [17]. If $\theta(u, v) = \phi$ for some pair of vertices u and v, we define the cycle connectivity between u and v to be 0. *Cycle connectivity* of G is defined as $CC(G) = Max\{C^G_{u,v} : u, v \in \sigma^*\}$. That is, cycle connectivity of a fuzzy graph G is defined as the maximum cycle connectivity of different pairs of vertices of G. A vertex w in a fuzzy graph is called a *cyclic cutvertex* if $CC(G - w) < CC(G)$. An edge uv of a fuzzy graph is called a *cyclic bridge* if $CC[G - (u, v)] < CC(G)$ [18]. A fuzzy graph G is said to be *cyclically balanced* if G has no cyclic cutvertices and cyclic bridges [18].

In [15] the edges of a fuzzy graph are classified as α-strong, β-strong and δ-edges. An edge xy of a fuzzy graph G is called *α-strong* if $\mu(xy) > CONN_{G-xy}(x, y)$. It is called *$\beta$-strong* if $\mu(xy) = CONN_{G-xy}(x, y)$ and a *δ-arc* if $\mu(xy) < CONN_{G-xy}(x, y)$. It is shown that α-strong edges are precisely the bridges of the fuzzy graph and hence

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