ARTICLE IN PRESS

FSS:7334

ELSEVIER

Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems ••• (••••) •••–•••

FUZZY sets and systems

www.elsevier.com/locate/fss

A categorical approach to minimal realization for a fuzzy language

S.P. Tiwari [a,*], Vijay K. Yadav [a], Bijan Davvaz [b], Renu [a]

[a] *Department of Applied Mathematics, Indian Institute of Technology (ISM), Dhanbad-826004, India*
[b] *Department of Mathematics, Yazd University, 89195-741, Yazd, Iran*

Received 4 June 2016; received in revised form 16 August 2017; accepted 1 December 2017

Abstract

In this study, we provide a general theory of minimal realization for a given fuzzy language in a category theory setting. We introduce categorical characterizations of a run map, reachability map, and observability map for a crisp deterministic fuzzy automaton. Finally, we show that all minimal realizations for a given fuzzy language are isomorphic to the Nerode realization for the same case.

Keywords: Crisp deterministic fuzzy automaton; Fuzzy language; Minimal realization

1. Introduction

Since the theory of fuzzy sets was introduced by Zadeh, fuzzy automata and languages have been studied as methods for bridging the gap between the precision of computer languages and vagueness. These studies were initiated by Santos [41], Wee [44], and Wee and Fu [45], and further developed by [19,20,29,33]. Fuzzy automata and languages with membership values in different structures have attracted considerable attention from researchers in this area (see [8,13–18,22,23,25,26,37,38,46,47]). In particular, Qiu [37,38] first established a fundamental framework for fuzzy automata and languages with membership values in a complete residuated lattice, which was further developed by [13, 14,46–48]. The concept of a crisp deterministic fuzzy automaton (a deterministic automaton equipped with a fuzzy set of final states) was introduced and studied by [8,13,25]. The importance of this automaton is supported by the fact that: (i) a fuzzy language is accepted by some fuzzy automaton iff it is accepted by some crisp deterministic fuzzy automaton (see [8,25]), and (ii) the Nerode automaton of a fuzzy automaton is a crisp-deterministic fuzzy automaton that is equivalent to a given fuzzy automaton (see [13]). In addition, in terms of applications, fuzzy automata are useful for intelligent interface design, clinical monitoring, neural networks, and fuzzy discrete-event systems (see [10,39,40] and the references therein).

* Corresponding author.
E-mail addresses: sptiwarimaths@gmail.com (S.P. Tiwari), vkymaths@gmail.com (V.K. Yadav), bdavvaz@yahoo.com (B. Davvaz), renuismmaths@gmail.com (Renu).

https://doi.org/10.1016/j.fss.2017.12.002

Please cite this article in press as: S.P. Tiwari et al., A categorical approach to minimal realization for a fuzzy language, Fuzzy Sets Syst. (2017), https://doi.org/10.1016/j.fss.2017.12.002

One of the most significant branches of the algebraic theory of languages and automata is Myhill–Nerode's theory [34,35], where formal languages and deterministic automata are studied based on their right congruences and congruences on free monoid. These right congruences on a free monoid are useful for the construction of a minimal deterministic automaton that can recognize a given language. Following the introduction of fuzzy automata and fuzzy languages, Myhill–Nerode's theory was also used to construct a minimal crisp deterministic fuzzy automaton for a given fuzzy language [14]. Furthermore, the concept of a derivative of fuzzy languages was used for this type of construction by [14]. The concept of minimization for fuzzy automata is also well known (see [6,9,21,33,36]), where the basic idea of minimization involves reducing the number of states for a fuzzy automaton by computing and merging indistinguishable states. However, the use of the term minimization in previous studies did not refer to the usual construction of the minimum set for all fuzzy automata that recognize a given fuzzy language, but instead it was simply a procedure for computing and merging indistinguishable states, which does not necessarily yield a minimal fuzzy automaton (see [14]).

Certain ideas, concepts, and tools from category theory are useful in the development of many aspects of theoretical computer science. In particular, many studies have considered a categorical approach to automata theory (e.g., [2–5, 7,11,12,42,43]). A fairly general definition of a "machine" in a category was introduced by [3], which as particular cases includes linear systems (used in control theory), standard automata with output, tree automata, and stochastic machines. An interesting problem in this context is the *realization problem*, which states that given a *behavior*, can we design a machine that realizes it. This problem was studied in a fairly general category theory setting by Goguen [12] and Arbib and Manes [3]. However, an analogous minimal realization could not be provided for stochastic automata [28]. In the present study, we provide a general theory of minimal realization for a given fuzzy language in general category theory setting.

Similar to automata theory, the usefulness of category theory in the study of fuzzy automata has been demonstrated in previous studies by [1,24,30–32,48]. In particular, Močkőr [30] provided a method for reducing fuzzy automata that is analogous to a method for the minimization of classical Moore type automata. The categorical method and filter theory in a lattice were used for a uniform treatment (or comparisons) of various definitions of fuzzy automata [31, 32], but the fuzzy automata addressed had no initial and final states, and thus their behaviors were not considered. In addition, Xing and Qiu [48] studied the categorical issue for L-valued automata with L-valued initial and final states over a complete residuated lattice, and they established the relationship between the category of L-valued automata and the category of non-deterministic automata. In the present study, we use concepts from category theory to study a minimal realization for a given fuzzy language. We introduce categorical characterizations of a run map, reachability map, and observability map for a crisp deterministic fuzzy automaton. Finally, we show that all minimal realizations for a given fuzzy language are isomorphic to the Nerode realization.

The remainder of this paper is organized as follows. In Section 2, we recall various concepts related to category theory. In Section 3, we consider the concepts associated with bipointed sets, crisp deterministic fuzzy automata, and fuzzy languages. In Section 4, we introduce a category and provide categorical interpretations of some concepts for a given crisp deterministic fuzzy automaton. In Section 5, we introduce the minimal realization for a fuzzy language in a category theory setting and we show that all minimal realizations for a fuzzy language are isomorphic to the Nerode realization. In the final section, we give some concluding remarks.

2. Preliminaries

In this section, we give some definitions and notations from category theory that we use in the following. We recall some of the basic categorical concepts employed in this study. The standard references for these concepts are [2,4,27].

Definition 2.1. A **category C** comprises:

(i) a class of **objects** (or **C**-objects);
(ii) for each pair A and B of **C**-objects, a set $\mathbf{C}(A, B)$ with members called **morphisms** (or **C**-morphisms) with **domain** A and **codomain** B, $f \in \mathbf{C}(A, B)$ is written as $f : A \to B$ or $A \xrightarrow{f} B$;
(iii) for each **C**-object A, a morphism $id_A : A \to A$ is called the **identity morphism** on A;
(iv) a "composition law" associated with each pair of **C**-morphisms $f : A \to B$ and $g : B \to C$, a **C**-morphism $g \circ f : A \to C$ is called a **composition** of f and g;

Please cite this article in press as: S.P. Tiwari et al., A categorical approach to minimal realization for a fuzzy language, Fuzzy Sets Syst. (2017), https://doi.org/10.1016/j.fss.2017.12.002