ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

An integrated fitting and fairing approach for object reconstruction using smooth NURBS curves and surfaces

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A R T I C L E I N F O

Article history:
Received 15 November 2017
Received in revised form 24 June 2018
Accepted 3 July 2018
Available online xxxx

Keywords:
Object reconstruction
NURBS curves and surfaces
Integrated fitting and fairing approach

A B S T R A C T

This paper presents a new approach for object reconstruction by means of smooth NURBS curves and surfaces. Compared to the common object reconstruction algorithms that first, fit a curve or surface to a dataset and then, try to make it smooth in a post-processing fairing stage, this article proposes to apply the fitting and fairing procedures simultaneously to achieve desirable results. In the integrated fitting and fairing approach, the respective fitting and fairing objectives are met in a multi-objective optimization process. In the developed methodology, the structure of the reconstructed curves and surfaces (e.g. the arrangement of the knot vectors and location of control points) is optimized in such a way that the desired fairness will be locally achieved in every segment of the reconstructed object. The functionality of the developed method is investigated by some industrial case studies.

1. Introduction

Object reconstruction from a set of data points is an applicable field in the area of reverse engineering. There are several articles in the literature concerning the creation of geometric models of curves and surfaces from measured data points [1–5]. Inasmuch as many real-world objects have been constructed by free-form curves and surfaces, the non-uniform rational B-splines (NURBS) appear to be a universal class for curve/surface fitting in computer-aided design, manufacture and engineering (CAD/CAM/CAE) [6–11]. There are many advantages to use the parametric representation of NURBS instead of the implicit forms which are deeply studied in [12].

Creating visually pleasing or so-called fair curves and surfaces is of vital importance in geometric modeling and object reconstruction, especially where the aesthetic aspects are decisive (for example in automotive bodies). Some useful methods dealing with mathematical definition of fairness for parametric curves and surfaces are introduced in [13]. One of the first methods in the literature regarding fairing B-spline curves is the knot removal–reinsertion approach presented by Farin et al. [14]. Their method, which focuses on the usefulness of curvature as a measure for curve fairness, is then extended to fairing the free-form surfaces [15,16]. Considering that B-splines are piecewise continuous curves, they may represent some undesirable curvature jump at knots. The knot removal–reinsertion algorithm first identifies those knots that make the large jump in the curvature, and then fairs the curve by removing and reinserting the knots so that the faired curve has the same structure as the initial one. However, this method reveals few shortcomings as highlighted in [17]. Among the other fairing algorithms, the target curvature driven fairing for planar B-spline curves presented by Li et al. [18] can be cited. In this approach, the target curvature plots, prescribed by the designers according to design intent, are used to identify bad

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https://doi.org/10.1016/j.camwa.2018.07.007
0898-1221/

points and bad curve segments. The corresponding control points are then modified using a constrained optimization. The local energy fairing of B-spline curves, presented by Eck and Hadenfeld [19], can be mentioned as another fairing algorithm which is also extended to the surface case [20]. The main idea of this approach is to minimize the integral of the square of the kth derivative of a given curve by modifying the control point that is expected to reduce the energy integral most significantly at each iteration. However, as it is proved by Lee [21], the fairness criterion should be expressed in terms of the arc-length parameter to achieve desirable results. Moreton and Séquin [22] proposed a fairness metric based on the variation of the curvature. They minimized the integral of the squared magnitude of the arc-length derivative of curvature to construct minimum variation curves (MVC). This integral evaluates to zero for circular arcs and straight lines that are inherently fair. They also extended their method to construct minimum variation surfaces (MVS). The MVC fairness metric will then be improved in the following sections of this article to construct fair curves and surfaces.

It is interesting to note that all aforementioned fairing algorithms apply a post-processing fairing approach to a given curve or surface which might be constructed in a prior stage. Such a curve/surface already has its approximate final shape but may have some aesthetic imperfections. In this case, generally, the fair curve/surface is not allowed to change its shape too much during the fairing process. However, to reconstruct a fair object from measured data points, the curve/surface fitting and fairing processes should be employed simultaneously to achieve desirable results.

In this paper, an improved object reconstruction approach is developed in which the fitting and fairing procedures are integrated and the respective objectives are met in a multi-objective optimization process. For this purpose, we consider the curve case as a preliminary investigation for the surface case. To be more precise, for reconstructing a surface from a point cloud, the first order of business is to fit the sectional parametric NURBS curves to the respective sectional data points with minimized error in such a way that the desired fairness (i.e. smoothness) is achieved for all curves. Once the sectional curves are faired, the skinning approach will be performed to reconstruct fair surfaces. Finally it should be pointed out that NURBS curves are piecewise continuous curves with a desired number of knot spans. Choosing the optimal positions of knots are of central importance in both fitting and fairing processes. However, considering the knots as unknown variables makes the optimization procedure nonlinear. For solving such an optimization problem, the particle swarm optimization (PSO) will be employed. More details about the proposed methodology will be described in the following sections. Section 2 expresses a short definition of NURBS curves and surfaces. In Section 3, the object reconstruction methodology is introduced in which the fitting and fairing procedures are also described. Section 4 illustrates the applicability of the proposed approach by means of some industrial case studies followed by Section 5 that discusses the results. Finally, Section 6 draws some conclusions.

2. NURBS curves and surfaces: Definition

In this section, the definition of NURBS curves and surfaces as well as the fairness metrics will be reviewed as a preliminary to the object reconstruction phase.

2.1. NURBS curves

Starting with the definition of the curves, a NURBS curve of degree p, as a piecewise continuous function, with $n+1$ control points $\boldsymbol{P}_i = \left[P_{ix}, P_{iy}, P_{iz}\right]$ and corresponding non-negative weights w_i is expressed as follows [12]:

$$\boldsymbol{C}(u) = \sum_{i=0}^{n} R_{i,p}(u)\boldsymbol{P}_i = \left[R_{0,p}(u) \quad R_{1,p}(u) \quad \ldots \quad R_{n,p}(u)\right]\begin{bmatrix}\boldsymbol{P}_0\\ \boldsymbol{P}_1\\ \vdots\\ \boldsymbol{P}_n\end{bmatrix} \tag{1}$$

where $R_{i,p}(u)$ as the ith basis function of degree p is:

$$R_{i,p}(u) = \frac{N_{i,p}(u)\, w_i}{\sum_{j=0}^{n} N_{j,p}(u) w_j} \tag{2}$$

In the above equations, $\boldsymbol{C}(u) = \left[C_x, C_y, C_z\right]$ is a vector-valued function whose components are represented separately as explicit functions of the curve parameter u over the knot vector $\boldsymbol{U}$ with $n-p+1$ non-zero knot spans. In addition, $N_{i,p}(u)$ is the ith B-spline basis function of degree p defined by the Cox–de Boor recursion formula [12]:

$$\boldsymbol{U} = [\underbrace{0, 0, \ldots, 0}_{p+1}, \bar{u}_{p+1}, \bar{u}_{p+2}, \ldots, \bar{u}_n, \underbrace{1, 1, \ldots, 1}_{p+1}] \tag{3}$$

$$\begin{aligned} N_{i,0}(u) &= \begin{cases} 1 & \bar{u}_i \le u < \bar{u}_{i+1} \\ 0 & \text{otherwise} \end{cases} \\ N_{i,p}(u) &= \frac{u-\bar{u}_i}{\bar{u}_{i+p}-\bar{u}_i} N_{i,p-1}(u) + \frac{\bar{u}_{i+p+1}-u}{\bar{u}_{i+p+1}-\bar{u}_{i+1}} N_{i+1,p-1}(u) \end{aligned} \tag{4}$$

For example, Fig. 1 depicts a 2D cubic NURBS curve with 7 control points and associated weights reported in Table 1. It is also assumed that the knot vector has uniformly-spaced mid-knots as $\boldsymbol{U} = [0, 0, 0, 0, 0.25, 0.5, 0.75, 1, 1, 1, 1]$. The control