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An iterative method for obtaining the Least squares solutions of quadratic inverse eigenvalue problems over generalized Hamiltonian matrix with submatrix constraints[☆]

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ABSTRACT

In this paper, we consider a class of constrained matrix quadratic inverse eigenvalue problem and its optimal approximation problem. It is proved that the proposed algorithm always converge to the generalized Hamiltonian solutions with a submatrix constraint of Problem 1.1 within finite iterative steps in the absence of roundoff error. In addition, by choosing a special kind of initial matrices, it is shown that the minimum norm solution of Problem 1.1 can be obtained consequently. At last, for a given matrix group in the solution set of Problem 1.1, it is proved that the unique optimal approximation solution of Problem 1.2 can be also obtained. Some numerical results are reported to demonstrate the efficiency of our algorithm.

1. Introduction

The second order differential system

$$A\ddot{x}(t) + B\dot{x}(t) + Cx(t) = f(t), \tag{1.1}$$

with mass, damping and stiffness matrices arises in the acoustic simulation of pro-elastic materials, the elastic deformation of anisotropic materials and finite element discretization in structural analysis [1–3]. The eigenvalues λ and the corresponding eigenvectors x of the quadratic eigenvalue problem

$$Q(\lambda)x := (\lambda^2 A + \lambda B + C)X = 0, \tag{1.2}$$

can interpret the dynamical behaviour of the second order differential system (1.1). The quadratic inverse eigenvalue problem is to construct the matrices A, B and C such that the quadratic eigenvalue problem (1.2) holds. That is, for given $X = (x_1, x_2, \ldots, x_n) \in \mathbb{C}^{n\times m}$ and $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m) \in \mathbb{C}^{m\times m}$, finding $A, B, C \in \mathbb{C}^{n\times n}$ such that

$$AX\Lambda^2 + BX\Lambda + CX = 0. \tag{1.3}$$

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If $A=0$, then matric quadratic inverse eigenvalue problem (1.3) reduces to matric generalized inverse eigenvalue problem

$$BX\Lambda+CX=0. \tag{1.4}$$

If $A=0, B=I$, then matric quadratic inverse eigenvalue problem (1.3) reduces to matric inverse eigenvalue problem

$$X\Lambda+CX=0. \tag{1.5}$$

If $X = x,\ \Lambda = \lambda$, then matric generalized inverse eigenvalue problem (1.4) reduces to generalized inverse eigenvalue problem $\lambda Bx=-Cx$.

The special solution of generalized inverse eigenvalue problem and matrix equations has raised much interest among researchers due to the wide applications in engineering and scientific computation (e.g., [4–14]). Zhang et al. [15] considered the sufficient and necessary conditions for the inverse eigenvalue problem with Hermitian generalized Hamiltonian matrices. Dai and Liang [16] considered solving the generalized inverse eigenvalue problem for the (P,Q)-conjugate matrices and the associated approximation problem by using generalized singular value decomposition and canonical correlation decomposition. Gao et al. [17] proposed a direct method for generalized inverse eigenvalue problem with the reflexive or anti-reflexive coefficient matrices. Moghaddam et al. [18] proposed an algorithm for reconstructing penta-diagonal coefficient matrices of generalized inverse eigenvalue problem. Mo and Li [19] discussed the inverse eigenvalue problem $AX = XB$ for Hermitian and generalized skew-Hamiltonian matrices with a leading principle submatrix constraint using singular value decomposition and Moore–Penrose generalized inverse. Cai et al. [20] considered the generalized inverse eigenvalue problem and its optimal approximation problem over partially bisymmetric matrices. And in [21] they proposed an iterative method to obtain the least-squares solutions of generalized inverse eigenvalue problem over Hermitian–Hamiltonian matrices with a submatrix constraint.

Inspired by the work in [20,21], in this paper, we consider the following constrained matrix quadratic inverse eigenvalue problems:

Problem 1.1. Given $X\in\mathbb{C}^{n\times m}$, $\Lambda=\mathrm{diag}(\lambda_1,\lambda_2,\ldots,\lambda_m)\in\mathbb{C}^{m\times m}$, $s=(s_1,s_2,\ldots,s_p,n+1-s_p,\ldots,n+1-s_2,n+1-s_1)\in D_{2p,n}$, $t=(t_1,t_2,\ldots,t_q,n+1-t_q,\ldots,n+1-t_2,n+1-t_1)\in D_{2q,n}$, $u=(u_1,u_2,\ldots,u_r,n+1-u_r,\ldots,n+1-u_2,n+1-u_1)\in D_{2r,n}$, $A_p\in\mathbb{C}^{2p\times 2p}$, $B_q\in\mathbb{C}^{2q\times 2q}$ and $C_r\in\mathbb{C}^{2r\times 2r}$. Let $S_1=\{X|X[s|s]=A_p, X[\bar{s}|\bar{s}]\in GH^{(n-2p)\times(n-2p)}\}$, $S_2=\{X|X[t|t]=B_q, X[\bar{t}|\bar{t}]\in GH^{(n-2q)\times(n-2q)}\}$, $S_3=\{X|X[u|u]=C_r, X[\bar{u}|\bar{u}]\in GH^{(n-2r)\times(n-2r)}\}$, find $A^*\in S_1$, $B^*\in S_2$ and $C^*\in S_3$ such that

$$\|A^*X\Lambda^2+B^*X\Lambda+C^*X\|=\min\|AX\Lambda^2+BX\Lambda+CX\|.$$

Problem 1.2. Let S_E be the set of solutions of Problem 1.1. For given $\bar{A},\bar{B},\bar{C}\in\mathbb{C}^{n\times n}$, find $\widehat{A},\widehat{B},\widehat{C}\in S_E$, such that

$$\|\widehat{A}-\bar{A}\|^2+\|\widehat{B}-\bar{B}\|^2+\|\widehat{C}-\bar{C}\|^2=\min_{(A,B,C)\in S_E}\|A-\bar{A}\|^2+\|B-\bar{B}\|^2+\|C-\bar{C}\|^2.$$

The rest of this paper is organized as follows: In Section 2, by reformulating Problem 1.1 as its equivalent Problem 2.1, we present an iterative method to solve the constrained matrix quadratic inverse eigenvalue Problem 1.2. The convergence properties of the proposed algorithm are reported later; In Section 3, we discuss the solution of Problem 1.2; Some numerical results are reported in Section 4; The conclusions are given in Section 5 at last.

In our notation, let $R^{m\times n}$ and $C^{m\times n}$ be the sets of all real and complex $m\times n$ matrices, respectively. Let $A\in C^{m\times n}$, we write $Re(A)$, $Im(A)$, $\bar{A}$, A^T, A^H, $\|A\|$, A^{-1}, and $\mathcal{R}(A)$ to denote the real part, imaginary part, conjugation, transpose, conjugate transpose, Frobenius norm, inverse, and the column spaces of matrix A, respectively. For any matrix $A=(a_{ij})$, $B=(b_{ij})$, matrix $A\bigotimes B$ denotes the Kronecker product defined as $A\bigotimes B=(a_{ij}B)$. For the matrix $X=(x_1,x_2,\ldots,x_n)\in\mathbb{C}^{n\times n}$, $\mathrm{vec}(X)$ denotes the vec operator defined as $\mathrm{vec}(X)=(x_1^T,x_2^T,\ldots,x_n^T)^T\in C^{mn}$. Let $I_n=(e_1,e_2,\ldots,e_n)$ and $S_n=(e_n,e_{n-1},\ldots,e_1)$ be the $n\times n$ unit matrix and reverse unit matrix, respectively, where e_i denotes its ith column of unit matrix. Let $D_{p,n}=\{d=(d_1,d_2,\ldots,d_p): 1\le d_1<d_2<\cdots<d_p\le n\}$ denote the strictly increasing sequences of p elements from $1,2,\ldots,n$. For $s=(s_1,s_2,\ldots,s_p)\in D_{p,n}$, $t=(t_1,t_2,\ldots,t_q)\in D_{q,n}$, $u=(u_1,u_2,\ldots,u_r)\in D_{r,n}$, we assume that $E_s=(e_{s_1},e_{s_2},\ldots,e_{s_p})\in\mathbb{C}^{n\times p}$, $E_t=(e_{t_1},e_{t_2},\ldots,e_{t_q})\in\mathbb{C}^{n\times q}$, $E_u=(e_{u_1},e_{u_2},\ldots,e_{u_r})\in\mathbb{C}^{n\times r}$. Also, $A[s|t]$ stands for the submatrix of A determined by rows indexed by s and columns indexed by t. Moreover, the notation $A[\bar{s},\bar{t}]$ represents the submatrix of A determined by deleting rows indexed by s and columns indexed by t.

Let $ASOR^{m\times m}$ stand for the sets of all $m\times m$ antisymmetric orthogonal matrices, i.e.,

$$ASOR^{m\times m}=\{J|J^TJ=JJ^T=I_m, J=-J^T, J\in R^{m\times m}\}.$$

In the space $C^{m\times n}$, the inner product can be defined as

$$\langle A,B\rangle=Re[tr(A^HB)]. \tag{1.6}$$

Definition 1.1. Let $J\in ASOR^{n\times n}$ be given.

(1) Matrix $A\in C^{n\times n}$ is called a generalized Hamiltonian matrix if $JAJ=A^H$. The set of all $n\times n$ generalized Hamiltonian matrices is denoted by $GH^{n\times n}$.

(2) Matrix $A\in C^{n\times n}$ is called a generalized skew Hamiltonian matrix if $JAJ=-A^H$. The set of all $n\times n$ generalized skew Hamiltonian matrices is denoted by $GSH^{n\times n}$.