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Modeling and an immersed finite element method for an interface wave equation☆

Jinwei Bai [a], Yong Cao [a,*], Xiaoming He [b], Hongyan Liu [c], Xiaofeng Yang [d]

[a] Department of Mechanical Engineering & Automation, Harbin Institute of Technology(Shenzhen), Shenzhen, Guangdong 518055, PR China
[b] Department of Mathematics & Statistics, Missouri University of Science & Technology, Rolla, MO 65401, USA
[c] School of Mathematical Science, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, PR China
[d] Department of Mathematics, University of South Carolina, Columbia, SC, 29208, USA

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ABSTRACT

The electromagnetic field, which is governed by Maxwell's equation, plays a key role in plasma simulation. In this article, we first derive the interface conditions when we rewrite the interface Maxwell's equation, whose problem domain involves complex media such as objects of different materials, into a parabolic–hyperbolic type of interface model, which is a modified wave equation by adding a first order time derivative term due to the lossy medium. Based on the interface conditions and the existing bilinear immersed finite element space for the interface Poisson equation, we propose an immersed finite element method for the spatial discretization of this parabolic–hyperbolic interface equation on a Cartesian mesh independent of the interface. Then we use a second order finite difference method for the temporal discretization in order to develop the full discretization scheme. Compared with the unstructured body-fitting mesh which is needed by the traditional finite element method for interface problems, the Cartesian mesh independent of the interface will significantly benefit the plasma simulation in the electromagnetic field, especially the particle models (such as the particle-in-cell method). This method provides a new efficient way to study varying electromagnetic field with objects of different materials on a Cartesian mesh independent of the interface, hence builds a solid foundation for the further study on the motion of plasma in this electromagnetic field. Numerical examples are provided to demonstrate the features of the proposed method.

1. Introduction

Particle models have been developed and widely applied for different plasma simulation problems, such as the ion/hall thruster [1–3] and lunar surface charging problems [4–6]. Many existing works only consider the electrostatic case or the electromagnetic case with a fixed external magnetic field [7–9]. However, many realistic applications involve a varying electromagnetic field, such as the propagation of electromagnetic wave in plasma which is governed by Maxwell's equation [10–14]. Furthermore, objects of different materials often need to be considered in the simulation domain for

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* Corresponding author.
E-mail addresses: baijinwei@stu.hit.edu.cn (J. Bai), yongc@hit.edu.cn (Y. Cao), hex@mst.edu (X. He), hongyan_liu1528@163.com (H. Liu), xfyang@math.sc.edu (X. Yang).

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various realistic situations. Hence solving interface Maxwell's equation with complex media plays a key role in plasma simulation.

The finite diffidence time domain (FDTD) method was proposed to solve the Maxwell's equation based on the approximation of temporal and spatial derivatives by central difference on staggered grids [15]. Due to its convenience in implementation and many other features, it has been further improved and used for different applications [16–18]. In recent years, the FDTD method has also been developed for the dispersive media and interface problems [19–22].

On the other hand, the finite element time domain (FETD) method was developed to solve the Maxwell's equation or wave equation [23]. In recent years, the FETD method has also been developed for dispersive media [24–28]. Pre-conditioners were also developed for solving Maxwell's equation or wave equations more efficiently [29–33]. However, the unstructured body-fitting meshes, which needs to be constructed to fit the objects' surface in the traditional finite element methods, are not efficient for the particle simulation models. Therefore, a new finite element method, which can solve a non-trivial interface problem on Cartesian mesh, is in a great need for solving the interface Maxwell's equation and its simplification to the interface wave equation.

The immersed finite element (IFE) method [34–48] was developed for efficiently solving interface problems with structured meshes independent of the interface, such as a Cartesian mesh. The algebraic system arising from the IFE method is symmetric positive-definite, which is a critical property for employing many fast solvers. While minimizing the extra efforts to modify the traditional finite element packages, IFE methods can also easily deal with complex interface with an optimal order of accuracy. These features make the IFE methods competitive for providing an accurate field solver to the particle simulation models based on structured meshes independent of the interface.

Therefore, the IFE method has been incorporated into the particle-in-cell (PIC) method to result in the IFE–PIC method, in which the IFE method solves the electric field with a fixed Cartesian mesh and the PIC method handles the simulation particles in the Cartesian mesh [49–51,5,52]. The IFE–PIC method has been applied to study the plasma problems for ion-optical thruster and lunar surface charging [53–55,4,56], especially for non-trivial object surface problems. However, none of the existing IFE methods consider the interface Maxwell's equation for the field solving.

In this paper, we develop an immersed finite element method to solve a simplified interface Maxwell's equation on a Cartesian mesh independent of the complex interface. We first derive the new interface conditions when we simplify the interface Maxwell's equation into a parabolic–hyperbolic type of interface model, which is a modified wave equation by adding a first order time derivative term due to the lossy medium. Then we recall the existing IFE basis functions which satisfy the interface conditions. Based on these preparations, we propose an immersed finite element method for this parabolic–hyperbolic interface equation based on (1) an immersed finite element spatial discretization on a Cartesian mesh independent of the interface; (2) a second order finite difference scheme for the temporal discretization. This accurate and efficient method will lay a solid ground for the plasma simulation involving varying electromagnetic field.

This article is organized as follows: in Section 2 we present and rewrite the interface Maxwell's equation; in Section 3 we recall the corresponding immersed finite element space; in Section 4 we propose the spatial and the temporal discretization for solving the interface model; in Section 5 we provide three numerical examples for demonstrating the approximation capability and the applicability of the proposed method; in Section 6 we draw a conclusion.

2. Maxwell's equation

In this section, we will first briefly recall the interface Maxwell's equation. Then we will rewrite it into another format of interface wave equation, especially deriving the interface conditions of the new format, for the TM (transverse magnetic) and TE (transverse electric) waves respectively. It is found that the interface equations of TM and TE waves can be unified into a parabolic–hyperbolic type of interface model, and its interface conditions satisfy the requirements of the existing IFE method basis functions for an interface elliptic equation.

For an internal current distribution of **J**, the electromagnetic field in the linear medium of isotropic materials satisfies the Maxwell's equation [57,58]:

$$\begin{aligned} \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} &= \mathbf{J}, \\ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= 0, \\ \nabla \cdot \mathbf{B} &= 0, \\ \nabla \cdot \mathbf{D} &= \rho, \end{aligned} \tag{2.1}$$

where electric field intensity $\mathbf{E} = (E_x, E_y, E_z)$, magnetic field intensity $\mathbf{H} = (H_x, H_y, H_z)$, magnetic flux density $\mathbf{B} = (B_x, B_y, B_z)$, electric flux density $\mathbf{D} = (D_x, D_y, D_z)$, electric current density $\mathbf{J} = (J_x, J_y, J_z)$, ρ is the electric charge density, and t is the time variable.

For lossless media, we have

$$\begin{aligned} \mathbf{D} &= \varepsilon \mathbf{E}, \\ \mathbf{B} &= \mu \mathbf{H}, \end{aligned} \tag{2.2}$$