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A Monte-Carlo based approach for pricing credit default swaps with regime switching

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ABSTRACT

This paper considers the valuation of a CDS (credit default swap) contract. To find out a more accurate CDS price, we work on an extended Merton's model by assuming that the price of the reference asset follows a regime switching Black–Scholes model, and moreover, the reference asset can default at any time before the expiry time. A general pricing formula for the CDS containing the unknown no default probability is derived first. It is then subsequently shown that the no default probability is equivalent to the price of a down-and-out binary option written on the same reference asset. By simulating the Markov chain with the Monte-Carlo technique, we obtain an approximation formula for the down-and-out binary option, with the availability of which, the calculation of the CDS price becomes straightforward. Finally, some numerical experiments are conducted to examine the accuracy of the approximation approach as well as the impacts of the introduction of the regime switching mechanics on the CDS price.

1. Introduction

In recent years, the explosive growth in the trading of financial derivatives has caused huge demand for the effective management of credit risks. This has prompted the fast development of credit derivatives. Among the most popular types of credit derivatives, credit default swaps (CDSs) are widely traded around the world as they are the basis of many other credit derivatives. It is pointed out by Ueno and Baba [1] that the liquidity of the CDS market is much higher than the market of traditional bonds written on the same reference asset because the principal is not traded for CDS contracts.

A CDS is actually a financial contract that involves two parties, namely, its buyer and seller. While the CDS buyer needs to make periodic payments to the seller until the expiry of the contract or a credit event occurs, its seller should compensate the buyer with the amount of the money specified in the contract in case of default. In other words, in return to all the protection fee paid by the buyer, the CDS seller needs to pay an amount, which is equivalent to the CDS notional face value minus the recovery value when the reference asset defaults. This regular fee that the buyer pays to the seller is usually referred to as the price of the CDS contract.

In the literature, there are two kinds of models for the valuation of CDSs. The models that belong to the first category are called the reduced-form credit risk models. These models are developed in [2–4], and further attract a number of authors, such as Lando [5], Madan and Unal [6]. This kind of models are usually mathematically appealing because the probability of default can be extracted from the market prices. However, one of the main drawbacks is that these models cannot capture the wide range of default correlations. On the other hand, the structural credit default models, as another alternatives, are

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able to provide correlation between different firms as they use the evolution of the asset price to determine the time when the default occurs. In this situation, the occurrence of a default event will be triggered if the value of the reference asset touches or drops below a certain level, which is called the default barrier. Merton [7] is believed to be the first to work on the structural model. He assumes that the reference asset follows a geometric Brownian motion and the default can occur only at the expiry of the CDS contract. It is obvious that both of the two basic assumptions are not appropriate. On one hand, the probability of default obtained by the Merton structural model is shown to be significantly less than the empirical default rate [8], and one possible reason is that the geometric Brownian motion does not incorporate any jump possibility [9]. One possible remedy is to introduce a jump component in the dynamics of the reference asset. For example, the geometric Brownian motion was replaced by a Poisson process in [10], while a jump–diffusion is adopted by Zhou [11]. On the other hand, the assumption that the default can only occur at the expiry is certainly unrealistic, as pointed out by a number of authors [12,13]. The setting that the default can occur at any time during the lifetime of the contract is indeed much more favored.

In this paper, the valuation of the CDS contract is considered under a regime switching Black–Scholes (B–S) model with the volatility of the asset allowed to jump among different states following a Markov chain. Our motivation originates from two aspects. One is that this is an alternative way to introduce jump components into the dynamics of the reference asset, and another one is that there is a lot of empirical evidence showing the existence of regime switching in real markets [14,15]. In addition, Merton's default mechanics is modified, so that in our model, the default can occur at any time during the lifetime of the CDS contract. Under these settings, we first derive a general closed-form formula for the price of the CDS contract containing the unknown no-default probability. An equivalence relationship between the unknown probability and a down-and-out binary option is then established. Afterwards, a two-step pricing approach based on the Monte-Carlo simulation is introduced to solve for the price of the binary option.

The remainder of the paper is organized as follows. In Section 2, the Merton structural model is extended to the regime switching model, and a general formula for the price of a CDS contract is derived. The equivalence between the only unknown term, the no-default probability, and the down-and-out binary option price is also established. Then, this particular kind of option price under the regime switching model is found with a two-step approximation approach. In Section 3, numerical examples and discussions are presented, followed by some concluding remarks in the last section.

2. Credit default swaps under the regime switching model

A CDS contract is a financial agreement between two parties with the buyer of the contract making a series of payments to the seller in order to receive a compensation when the reference asset belonging to the third party defaults. In this section, the pricing of a CDS contract will be considered under a regime switching framework. In particular, we shall first introduce the default model for the CDS contract. Then, a two step approximation method based on the Monte-Carlo simulation is designed and presented. We point out that the price of a swap refers to the spread, i.e., the regular fee that the buyer pays to the seller, instead of being its value as usual, and is often quoted as the ratio of the reference asset price.

2.1. The default model

As already pointed out in the introduction, Merton's assumption that the default can only occur at the expiry of the contract is not realistic. To be closer to practice, in this subsection, such an assumption will be modified and a general formula for the price of the CDS will be derived.

Unlike Merton, we now assume that the default can occur at any time during the lifespan of the CDS contract. To find out its price, it is necessary to analyze the cash flow of the contract. Let T be the expiry time, and the current time be 0. In addition, we denote $p(t)$ as the probability of no default before the time t. We have $p(0) = 0$ because the CDS contract becomes meaningless if the default occurs at the initial time. Moreover, let D and S_t be the default barrier and the value of the reference asset, respectively. It is clear that the default does not occur at the time t if $S_t > D$. Therefore, $p(t)$ can be expressed as

$$p(t) = P(\min_{0\leq s\leq t} S_s > D). \tag{2.1}$$

With the default probability at hand, we are now ready to analyze the cash flow between the CDS buyer and seller. As the CDS buyer should regularly pay the protection fee to the seller before the default occurs, it is not difficult to show that the expectation of the amount of the payment from the buyer to the seller between time t and $t + dt$ is $(cMdt)p(t)$ with c and M being the target spread per unit time and the face value of the reference asset, respectively. This implies that the present value of the cash flow of the buyer (denoted as V_1) is

$$V_1 = \sum_t [e^{-rt} cMp(t)dt] = cM \int_0^T e^{-rt} p(t)dt,$$

where r represents the risk-free interest rate. On the other hand, once a default occurs, the seller should pay $(1 - R)M$ to the buyer with R being the recovery rate specified in the contract. Since the probability of the default taking place between time t and $t + dt$ can be calculated as

$$[1 - p(t + dt)] - [1 - p(t)] = p(t) - p(t + dt) = -dp(t),$$