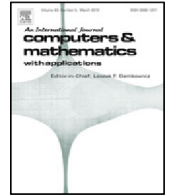




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# Analysis on the existence of the steady-states for an ecological–mathematical model with predator–prey-dependent functional response<sup>☆</sup>

Yunfeng Jia<sup>a,\*</sup>, Bimei Luo<sup>a</sup>, Jianhua Wu<sup>a</sup>, Hong-Kun Xu<sup>b</sup><sup>a</sup> School of Mathematics and Information Science, Shaanxi Normal University, Xi'an, Shaanxi 710062, China<sup>b</sup> School of Science, Hangzhou Dianzi University, Hangzhou, Zhejiang 310018, China

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## ABSTRACT

We establish a reaction–diffusion ecological–mathematical model with predator–prey dependent functional response and the Neumann boundary conditions. First, we study the bifurcation solution emitting from the unique positive constant solution by considering the local bifurcation. Finally, we mainly analyze the coexistence of the prey and predator with the help of the fixed point index theory, where we use a different approach in calculating one of the fixed point indices.

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## 1. Introduction

In the early twentieth century, the study of spatial ecology became an interesting and significant topic, whose main aim is to study the causes and the consequences of the distribution of species over time and space. Based on the excellent works of some ecologist and mathematicians, a great deal of ecological–mathematical models have been established, see [1–8] for examples. Among numerous ecological–mathematical models, the predator–prey model is a very important type, such as [9–14]. One of the important themes for both ecology and mathematics is the dynamical relationships between predators and their preys. For example, in [15], Alonso et al. studied how diffusion affects the stability of the model, and indicated the difference between ratio-dependent type and prey-dependent type models. It was showed that the prey-dependent models cannot lead to spatial structure driven instability through diffusion, however, the ratio-dependent models can. In [16], Pal et al. pointed out that the prey-dependent type models have been challenged since the fact that the rate of the predator consuming prey does not only depend on prey density but also depend on the density of predator itself. In fact, in many cases, especially when the predators search for food, they always have to share food when they compete for food. Consequently, some scholars think that the functional response in predator–prey models should be predator–prey-dependent. As results, some predator–prey-dependent functional responses and the corresponding mathematical models emerged successively, such as Hassell–Varley type [17,18], Beddington–DeAngelis type [16,19,20] and Crowley–Martin type [21,22] functional responses, these are the typical predator–prey dependent functional response.

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In [23], Chakraborty et al. studied the following predator–prey-dependent model with non-selective harvesting effort

$$\begin{cases} u_t = ru(1 - \frac{u}{k}) - \frac{\alpha uv}{a + bu + cv} - q_1 me u, \\ v_t = sv(1 - \frac{v}{l}) + \frac{\beta uv}{a + bu + cv} - q_2 me v, \end{cases} \tag{1.1}$$

where  $u$  and  $v$  are two fish species, and represent densities of prey and predator at time  $t$ , respectively;  $r$  and  $s$  are the intrinsic growth rate of  $u$  and  $v$ ;  $\alpha$  is the maximal relative increase of predation, and  $\beta$  represents the conversion efficiency of consumed prey into new predators; parameters  $a$  and  $b$  describe, respectively, the effects of capture rate and handling time on the feeding rate, and  $c$  describes the magnitude of interference among predators; the terms  $q_1 me u$  and  $q_2 me v$  stand for the non-selective harvesting for the prey and predator,  $q_1$  and  $q_2$  are catchability coefficients of prey and predator populations, respectively,  $e$  is the combined fishing effort used to harvest the population, and  $m$  is the fraction of the stock available for harvesting which means that it is possible to control the area of harvesting by considering a numerical value. All parameters are assumed to be positive. In model (1.1), it is assumed that the populations grow in a closed homogeneous environment and follow logistic type of growth. The authors discussed this model incorporating partial closure for the populations. Model (1.1) assumes that the species  $v$  feeds on species  $u$ , and the intraspecific competition within  $u$  exists, which leads to  $u$  compete with each other for using an external resource as food that helps themselves to grow. Thus, system (1.1) shows that, in addition to predation between these two fish species and the intraspecific competition in  $u$ , both the species are subjected to a combined and non-selective harvesting effort.

In order to consider the natural tendency of each species to diffuse to areas of smaller population concentration and the inhomogeneous distribution of the predator and prey in different spatial location within a fixed bounded domain at any given time, in [24], Wang et al. discussed the PDE model of reaction–diffusion type corresponding to (1.1). Specifically, they discussed the following system with Dirichlet boundary conditions:

$$\begin{cases} u_t - \Delta u = ru(1 - \frac{u}{k}) - \frac{\alpha uv}{a + bu + cv} - q_1 me u, & (x, t) \in \Omega \times (0, \infty), \\ v_t - \Delta v = sv(1 - \frac{v}{l}) + \frac{\beta uv}{a + bu + cv} - q_2 me v, & (x, t) \in \Omega \times (0, \infty), \\ u = v = 0, & (x, t) \in \partial\Omega \times (0, \infty), \\ u(x, 0) \geq, \neq 0, v(x, 0) \geq, \neq 0, & x \in \Omega, \end{cases} \tag{1.2}$$

where  $\Omega \subset R^N$  is an open and bounded domain with smooth boundary  $\partial\Omega$ . By using the boundedness of positive solutions and the global bifurcation theory, the authors found that the local bifurcation branch can be extended to the global bifurcation. Their results show that the predator–prey-dependent functional response and the non-selective harvesting factors both have critical roles on the dynamics of the model. The main conclusion indicates that the global solutions may exist if intrinsic growth rates of prey and predator are suitably dominated by system parameters. In addition, the PDE model of reaction–diffusion type corresponding to (1.1) with Neumann boundary conditions and without non-selective harvesting effort

$$\begin{cases} u_t - \Delta u = ru(1 - \frac{u}{k}) - \frac{\alpha uv}{a + bu + cv}, & (x, t) \in \Omega \times (0, \infty), \\ v_t - \Delta v = sv(1 - \frac{v}{l}) + \frac{\beta uv}{a + bu + cv}, & (x, t) \in \Omega \times (0, \infty), \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, & (x, t) \in \partial\Omega \times (0, \infty), \\ u(x, 0) \geq, \neq 0, v(x, 0) \geq, \neq 0, & x \in \Omega \end{cases} \tag{1.3}$$

is investigated in [25]. Where  $\frac{\partial}{\partial n}$  is the outward unit normal vector on  $\partial\Omega$ , the homogeneous Neumann boundary conditions indicate that the two fish species are self-contained and have no populations flux across the boundary  $\partial\Omega$ . A no-flux boundary condition is composed on  $\partial\Omega$  so that the ecosystem is closed to the exterior environment. In [25], the author’s main concern is the large time behaviors of the nonconstant solutions and the local and global asymptotic stability of the positive constant solution. Some numerical examples support the main results.

In this paper, we continue to focus on the model (1.3), but the difference lies in that our main aim now is the existence of the steady-states corresponding to (1.3).

This paper is organized as follows. In Section 2, we give some preliminaries which will be used in the subsequent sections, including the existence and uniqueness of the positive constant solution, the global attractor and the persistence property. In Section 3, we study the bifurcation solution emitting from the positive constant solution by considering the local bifurcation. And in Section 4, we analyze the coexistence of the prey and predator with the help of the fixed point index theory and by using a new idea, which is our main concern of this paper.

**2. Preliminaries**

In this part, we give some preliminaries which will be used later. These are taken from reference [25].

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