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Computational homogenization of fresh concrete flow around reinforcing bars

F. Kolařík *, B. Patzák, J. Zeman

Department of Mechanics, Faculty of Civil Engineering, CTU in Prague, Czech Republic

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ABSTRACT

Motivated by casting of fresh concrete in reinforced concrete structures, we introduce a numerical model of a steady-state non-Newtonian fluid flow through a porous domain. Our approach combines homogenization techniques to represent the reinforced domain by the Darcy law with an interfacial coupling of the Stokes and Darcy flows through the Beavers-Joseph-Saffman conditions. The ensuing two-scale problem is solved by the Finite Element Method with consistent linearization and the results obtained from the homogenization approach are verified against fully resolved direct numerical simulations.

1. Introduction

The motivation for this work comes from the computational modeling of *self-compacting concrete* (SCC) — a type of high-performance concrete developed in the late nineties in Japan to produce more durable structures [1]. The increased performance is ensured by the fact that concrete casting is driven purely by self-weight without the need for vibration casting, which is convenient especially for highly reinforced structures with limited space between reinforcing bars [2]. In comparison to the conventional concretes that are designed primarily for their compressive strength, SCCs must meet additional rheological requirements, such as higher liquidity, in order to ensure that the mix fills the whole form-work at low risk of phase segregation. For this reason, the focus of the numerical modeling of SCC is not only on the structural, but also on the casting performance, and thus it relies on techniques of computational fluid mechanics.

Depending on the level of detail, different phenomena can be taken into consideration when modeling fresh concrete flow, e.g. [3–5]. In the most realistic case, fresh concrete is considered as a suspension of *interacting particles convected by a fluid*. These models can be treated numerically by discrete particle schemes, such as the discrete element method [6] and smoothed particle hydrodynamics [7,8], or by fluid solvers coupled to particle-tracking algorithms [9]. However, the major disadvantage of such simulation tools is their applicability only to material- or laboratory-scale tests, due to computational demands of the detailed resolution.

The constitutive models aiming at structural-scale applications consider concrete as a *homogeneous* non-Newtonian *fluid*, whose rheological properties are derived from the mix composition, e.g. [10–12]. The concrete flow can be then efficiently simulated using the Finite Element Method (FEM) in the Lagrangian [13,14] or in the Eulerian [15] setting. Of course, this efficiency comes at the cost of a coarser description of the flow. Consequently, sub-scale phenomena can only be accounted for approximately by post-processing simulation results, e.g., to determine the distribution and orientation of reinforcing fibers [16], or by heuristic modification of constitutive parameters, e.g. to account for the effect of traditional reinforcement [17]. Especially the latter aspect is critical in the modeling of casting processes in highly-reinforced structures, which represent the major field of application for SCC.

In this paper, we propose an efficient approach which incorporates the effects of traditional reinforcement on fresh concrete flow. The tools of computational homogenization, e.g. [18–20], will be utilized to avoid the need to resolve flows around each reinforcing bar, which would lead to excessive simulation costs comparable to those of the particle-based models. To this purpose, the structure is decomposed into three parts:

- *reinforcement-free* zone occupied by a homogeneous non-Newtonian fluid,
- *reinforced* zone where a two-scale homogenization scheme is employed, and

* Corresponding author.
E-mail address: kolarfil@gmail.com (F. Kolařík).

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- homogenization-induced *interface* separating the reinforced and reinforcement-free zones.

As the first step, we restrict ourselves to *steady state* flows; an extension to the transient case will be reported separately following the framework introduced in [15].

In the reinforced domain, we will assume that the reinforcing bars are rigid, acting as obstacles to the flow, and that their size (micro-scale) is small compared to a characteristic size of the structure or of the concrete form-work (macro-scale). It now follows from the results of mathematical homogenization theory, namely by Sanchez-Palencia [21, Chapter 7], Tartar [22], and Allaire [23] for Newtonian fluids and by Bourgeat and Mikelić [24,25] for non-Newtonian fluids (see also [26] for an overview), that the flow in this region can be accurately approximated by a *homogeneous Darcy law*. The relation between the macro-scale pressure gradient and the seepage velocity is defined implicitly, via a micro-scale boundary value problem that represents a Stokes flow in the representative volume element (RVE) of the reinforcing pattern, driven by the gradient of the macro-scale pressure. For the numerical treatment of the ensuing two-scale model, we will rely on the variationally-consistent approach developed recently by Sandström and Larsson [27] and Sandström et al. [28], which combines the variational multi-scale method [29] with first-order computational homogenization [18,20].

As a result of the homogenization procedure, an artificial interface appears that separates the Stokes domain from the Darcy domain. In order to couple the flows in both domains, we will employ the *Beavers-Joseph-Saffman conditions* [30,31] that effectively act as frictional conditions to the Stokes flow. The ensuing interface constants, relating the traction vector and the relative tangential slip in velocity, can be estimated from an auxiliary boundary value problem at the cell level, derived for Newtonian fluids by a refined asymptotic analysis in the seminal work of Jäger and Mikelić [32] and verified later by direct numerical simulations for free laminar flow by Jäger et al. [33] and Carraro et al. [34].

Following these considerations, the rest of the paper is organized as follows. In Section 2, we present the development of the homogenized model, including the variationally-consistent homogenization in the Darcy domain and a discussion of the Stokes-Darcy coupling. The numerical aspects of the problem are gathered in Section 3. In Section 4, we address the errors introduced by the homogenization and the coupling procedures, by comparing the homogenized model with fully resolved simulations. The potential of the developed model is critically discussed in the concluding Section 5, where we highlight the need for a more refined interface description.

The novelty of this paper is twofold. We see our first contribution in the development of a systematic procedure to incorporate the effect of reinforcement into homogeneous models, thereby *rationalizing the porous media analogy* introduced by Vasilić et al. [17] on heuristic grounds. The second novelty lies in our treatment of the homogenized model and the Stokes-Darcy coupling *simultaneously*. Indeed, there are several studies on the Stokes-Darcy coupling with the help of proper interface conditions, e.g. [35–37], in which the permeability is given in advance instead of being upscaled from the underlying micro-structure. Other works deal with the homogenization of the Stokes flow through the porous domain, e.g. [28,27], or consider the coupled Stokes-Darcy flow, but do not allow for the flow between the domains, e.g. [33,34]. This study seems to be the first one considering these effects simultaneously for both Newtonian and non-Newtonian fluids. In these aspects, the present work extends our recent contribution [38] where only linear Newton rheology was considered and where the methodology was explained is much less detail.

2. Formulation of the problem

In this section, we present the derivation of the homogenized model, employing the framework of the variationally consistent homogenization [27,28] in the bulk and the refined asymptotic analysis of Jäger and Mikelić [32] at the internal interface. For the sake of notational simplicity, we restrict ourselves to the two-dimensional setting shown in Fig. 1 and refer the readers interested in the general case to [27,28]. Specifically, the obstacles are assumed to be arranged according to a regular grid of cells, further referred to as Volume Elements (RVEs), and to be located symmetrically with respect to the center of each RVE without intersecting its boundary. Further, the perforated domain is assumed to be fully contained within the unperformed domain, where the external boundary conditions are imposed.

2.1. Strong form

As a point of departure, consider the Stokes flow over a perforated domain as shown in Fig. 1. We denote, in agreement with Fig. 1, Ω_F as the reinforcement-free part of the domain and Ω_P as a part of the domain with the obstacles (further called perforated sub-domain). Boundary of the obstacles is denoted as $\partial\Omega_P$, while the outer boundary $\partial\Omega_F$ is split into two disjoint parts $\partial\Omega_F^p$ and $\partial\Omega_F^u$ corresponding to the type of applied boundary condition; Γ stands for the interface between the perforated, Ω_F, and the unperforated, Ω_P, domains. By $\boldsymbol{n}$, we denote both the outer unit normal vector to $\partial\Omega_F$ and Γ, in the latter case pointed from to Ω_P to Ω_F.

The governing equations of the steady-state flow of an incompressible fluid in the union of domains $\Omega_F \cup \Omega_F$ take the form

$$-\nabla\cdot\boldsymbol{\tau}(\boldsymbol{D}(\boldsymbol{u})) + \nabla p = \rho\boldsymbol{b} \qquad \text{in } \Omega_P\cup\Omega_F \tag{1a}$$
$$\nabla\cdot\boldsymbol{u} = 0 \qquad \text{in } \Omega_P\cup\Omega_F \tag{1b}$$
$$\boldsymbol{u} = \boldsymbol{0} \qquad \text{in } \partial\Omega_P \tag{1c}$$
$$(\boldsymbol{\tau} - p\boldsymbol{I})\cdot\boldsymbol{n} = -\hat{p}\boldsymbol{n} \qquad \text{on } \partial\Omega_F^p \tag{1d}$$
$$\boldsymbol{u} = \hat{u}_n\boldsymbol{n} \qquad \text{on } \partial\Omega_F^u. \tag{1e}$$

Our notation is standard; $\boldsymbol{\tau}$ stands for the deviatoric part of a stress tensor, the strain rate tensor $\boldsymbol{D}$ is obtained as the symmetrized gradient of the unknown velocity field $\boldsymbol{u}$,

$$\boldsymbol{D}(\boldsymbol{u}) = \frac{1}{2}\left(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right),$$

p denotes pressure, $\rho\boldsymbol{b}$ are body forces, $\boldsymbol{I}$ is a unit second order tensor and $\bar{u}_n$ and $\hat{p}$ refer to the boundary data. Notice that we set the velocity on the boundary of the bars $\partial\Omega_P$ to zero in (1c). Another physically reasonable possibility is to consider zero velocity in the normal direction to the boundary of the bars only. However, the former case is preferable from the numerical point of view and also it is frequently used by others [27]. We believe that in the most situations it is also more realistic, because in real castings the concrete will stick to the bars during the flow.

2.2. Variational form and two-scale decomposition

The weak form of (1) amounts to finding a pair $(\boldsymbol{u}, p)$ such that

$$\int_{\Omega_P}\nabla\boldsymbol{\delta w} : \boldsymbol{\tau}(\boldsymbol{D}(\boldsymbol{u}))\,\mathrm{d}x - \int_{\Omega_P}(\nabla\cdot\boldsymbol{\delta w})p\,\mathrm{d}x + \int_{\Omega_P}\delta q(\nabla\cdot\boldsymbol{u})\,\mathrm{d}x \tag{2a}$$
$$+\int_{\Omega_F}\nabla\boldsymbol{\delta w} : \boldsymbol{\tau}(\boldsymbol{D}(\boldsymbol{u}))\,\mathrm{d}x - \int_{\Omega_F}(\nabla\cdot\boldsymbol{\delta w})p\,\mathrm{d}x + \int_{\Omega_F}\delta q(\nabla\cdot\boldsymbol{u})\,\mathrm{d}x \tag{2b}$$
$$+\int_{\partial\Omega_F}\boldsymbol{\delta w}\cdot\hat{p}\boldsymbol{n}\,\mathrm{d}s + \int_\Gamma[\![\boldsymbol{\delta w}\cdot\boldsymbol{\tau}\cdot\boldsymbol{n}]\!]\,\mathrm{d}s - \int_\Gamma[\![\boldsymbol{\delta w}\cdot p\boldsymbol{n}]\!]\,\mathrm{d}s \tag{2c}$$
$$=\int_{\Omega_P}\boldsymbol{\delta w}\cdot\rho\boldsymbol{b}\,\mathrm{d}x + \int_{\Omega_F}\boldsymbol{\delta w}\cdot\rho\boldsymbol{b}\,\mathrm{d}x \tag{2d}$$