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Analysis of the quasiperiodic response of a generalized van der Pol nonlinear system in the resonance zone

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ABSTRACT

The paper addresses the description of the complex behavior of simple nonlinear systems that are excited in the neighborhood of the resonance frequency. Depending on the detuning of the excitation frequency, resonant response can vary from purely stationary to various cases of quasiperiodic or chaotic response. This type of response is characterized by regular or irregular changes of the amplitude, which, in the quasiperiodic case, represents the beating effect. The beating frequency then changes from zero in resonance to a positive value outside the resonance zone. The ratio of the energy content of quasiperiodic and stationary components decreases in the same time. Starting at a certain detuning, the quasiperiodic component fully vanishes and the stationary component absorbs the whole response energy.

The motivation of this study originates from the aeroelasticity of large bridges, the tuned mass damper application, and other domains of civil engineering, where beating effects have been observed in the past. Such effects are very dangerous; hence, robust theoretical background for the design of adequate countermeasures should be developed. Nevertheless, investigations of the internal structure of a quasiperiod and its dependence on the difference between excitation frequency and eigenfrequency were conducted on a heuristic basis and an objective theoretical background is still missing.

A qualitative analysis of nonlinear systems using combinations of harmonic balance, small-parameter methods, and perturbation techniques is presented in the paper. Parametric evaluations are presented along with a discussion concerning the applicability of the presented approach.

1. Introduction

The response of a number of nonlinear dynamic systems under an additive harmonic excitation with a frequency located in the resonance zone and its vicinity is characterized by a tendency to induce the quasiperiodic response. There are two main components of this process: (i) auto-oscillations associated with the relevant eigenfrequency, and (ii) stationary forced vibrations. They can emerge either separately or in combination. The latter case results in a beating effect. Each of the components can be stable or unstable. The stable or unstable character determines the final shape of the system response. Consequently, the pattern of the resulting process is strongly dependent on the system parameters and mainly on the excitation amplitude and the difference between the excitation frequency and the eigenfrequency — detuning (see comments below Eq. (5)). This phenomenon can be observed in single-degree-of-freedom (SDOF) systems as well as in more complicated systems either with concentrated masses (multi-degrees of freedom – MDOF) or with continuously distributed parameters.

When the system's eigenfrequency and the frequency of excitation coincide, then the system response is stationary after the transient time elapses and the influence of initial conditions vanishes. However, provided the excitation frequency differs from the system's basic eigenfrequency by more than a very small threshold, various cases of quasiperiodic response can occur that have the character of a beating process. The period length of these beatings is infinite in resonance (i.e., the response is stationary) and slowly reduces with increasing detuning. The quasiperiodic character vanishes at a certain distance from the resonance zone, when the auto-oscillation component disappears and only forced vibration remains and represents the response as a whole.

Many outstanding papers investigating in general the resonance domain have been published during the last three decades. A few papers have dealt with the similar topic of quasiperiodicity and discussed theoretical aspects, e.g., [1,2]. Some studies have focused on the phenomenon itself on a phenomenological level [3,4], and summarize some methodological aspects [5]. Interesting studies have been conducted in the field of astrophysics [6] and in many

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other areas. Regarding the quasiperiodic process itself, the shape of the response envelope within one quasiperiod is a function of the "slow time." The process of quasiperiodicity is rather deterministic although it moves around the zero of the Lyapunov exponent. Hence, a temporary or permanent passage to a chaotic regime is also possible, (see, e.g., the famous monograph [7] and many additional papers discussing special aspects, for instance [8–10]). If the quasiperiodic process is plotted in the Poincaré map, we can see a large width attractor as a rule, along with various types of the post-critical response. The whole area of the quasiperiodic response is closely related with bifurcations of various types and therefore, our work refers to this theoretical background (see among others [11,12]).

Despite a wide variety of relevant studies conducted, a systematic investigation of the internal structure of a quasiperiod and its dependence on detuning is still lacking. Namely the analytical approaches investigating important parameter areas of these nonlinear systems are rare. The first partial attempt at describing the topic has been already published by the authors in [13] as regards the phenomenon of the beating in van der Pol operator-related systems with a SDOF. In the past, a study on spherical dynamic pendulum stability [14] had been conducted indicating an aspect of the beating phenomenon. Later, articles have been published [15,16] that demonstrate some theoretical basis of beating effects from an engineering perspective along with some recommendations for tuned mass damper (TMD) design. However, a detailed analysis of the response structure in the resonance zone is still missing and is important in order to recognize the applicability of the system from a practical point of view. The reason behind the theoretical investigations of the resonance zone and relevant quasiperiodic response lies in the need to protect systems against this phenomenon and, furthermore, in wide possibilities of sophisticated applications in practice in fields other than civil engineering.

The paper is organized as follows. In Section 2, two simple systems used in the discipline of civil engineering are introduced. They exhibit beating effects in the vicinity of the resonance, namely (i) the generalized van der Pol equation and (ii) the equation system describing the dynamic behavior of the spherical pendulum. The former case is studied in a greater detail in Section 3. Section 4 is devoted to a qualitative analysis of several special cases leading to particular types of nonlinear response. The theoretical results are illustrated using detailed plots and are numerically verified where appropriate throughout the paper.

2. Cases of quasiperiodic response

We can consider in general that many nonlinear systems are characterized by a quasiperiodic response (or beating effects) when the excitation frequency is close to the basic eigenfrequency (small detuning—the system is working in the resonance zone). This beating effect can be observed in systems with softening stiffness, small or zero linear damping, where the nonlinear part stabilizes the response process, etc. A couple of areas can be mentioned, where beating effects can occur due to nonlinearity in relevant differential operators. They were briefly outlined in Section 1 and will be also quoted later in this section, even though some of them consider the beating effects oppositely as a basic phenomenon of a device functionality.

Despite this fact, the authors of this paper gained the main motivation to study two areas of civil engineering, namely (i) the aeroelasticity of an SDOF system modeling a bridge deck section dynamics when tested in a wind channel (reduced flutter) and (ii) TMD effectiveness and reliability assessment. The dynamic response of both is often related with beating effects as experimental measurements and many numerical simulations show (see for instance [17–20]). These phenomena should be suppressed as much as possible. Basically, they significantly contribute to the reduction of system effectiveness or can result in a response beyond admissible limits (or collapse).

It is worth noting that the mechanism of creation of the beating or quasiperiodic effect differs in both the cases. The spherical pendulum is an auto-parametric system and the beating is caused by a periodical exchange of energy between both the components. The onset of the spatial response is interrelated with the stability loss of the planar solution. On the other hand, the quasiperiodic behavior of the van der Pol equation originates from detuning of the eigenfrequency and driving frequency of the system. However, it is found that the physical character of both and mainly the parameters that should be treated in order to suppress this phenomenon are very close as described later in this paper. Broadly speaking, the phenomenon of quasiperiodic response or beating effects can be expected everywhere in a nonlinear differential system either of SDOF or MDOF character with appropriate characteristics enabling an occurrence of self-excited effects. Combination of the system, which is prone to the self-excited behavior, and the external excitation with a frequency around any eigenfrequency can lead to these phenomena.

The example presented in Fig. 1 is the generalized van der Pol or Rayleigh oscillator given by equation

$$\ddot{u} - (\eta - \nu u^2 + \vartheta u^4)\dot{u} + \omega_0^2 u = P\omega^2 \cos \omega t, \qquad (1)$$

where rotation coordinate $\varphi(t) = 0$ is kinematically suppressed, $\omega_0^2 = K/m$ is the eigenfrequency of the associated linear system with stiffness K and concentrated mass m; η, ν, ϑ are coefficients of linear viscous and nonlinear damping; ω is the excitation frequency; $P\omega^2 = F\omega^2/m$ is the amplitude of the excitation force (per unit mass), and P can be interpreted as an eccentricity of a mass m rotating with a frequency ω or amplitude of pressure variation during vortex shedding. Eq. (1) characterizes the nonlinear vibration of an SDOF system modeling the reduced flutter as one of post-critical response types of an aeroelastic system (see the left part of Fig. 2). Coefficients η, ν, ϑ and their relation are responsible for the response portrait and solution stability; for more details, see, e.g., [21,22]. Their ratio decides about the existence of respective limit cycles. It can be shown that in aeroelasticity of systems modeled by Eq. (1), see, e.g., [9], there can exist one (stable) or two (one stable and one unstable) limit cycles. The latter case can imply a possibility of the collapse and the beating effect that make the system more sensitive to reach this state. Concerning the right side of Eq. (1), the vortex shedding is the origin of additive harmonic excitation. Unfortunately, the frequency of this excitation process is around the system eigenfrequency as a rule, and this has been

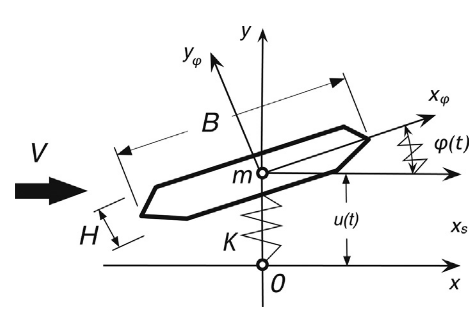

Fig. 1. Scheme of a bridge deck section in a wind channel modeled as the generalized van der Pol oscillator; the TDOF system is commonly used, and in the actual case only coordinate $u(t)$ is active (SDOF), while the rotation coordinate $\varphi(t)$ has been kinematically suppressed.

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