ARTICLE IN PRESS

Computers and Structures xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Stiffness of scarf joints with dowels

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ARTICLE INFO

Article history:
Accepted 3 March 2017
Available online xxxx

Keywords:
Timber structures
Joint
Beam
Pin
Key
Dowel

ABSTRACT

Repairs to historical timber structures include connecting existing beams and new beams. Older ways of making the connection need to be used to ensure authenticity of the beam. In most cases, e.g. floor structures, the second limit state – serviceability – is of decisive importance. The new connection softens the beam and increases the displacement. The most widely-used joint is a scarf joint with two or more bolts or dowels. It is not clear whether greater obliquity of its cheeks is desirable, or the effect of different numbers of bolts on the stiffness of the beam. In Germany, a scarf joint with four bolts is recommended, but there is no validation for this from the structural perspective. The behaviour of joints with two bolts or with four bolts seems to be the same. In the process of repairing historically valuable timber structures, the most widely-used joint is with wooden dowels. One pin can be combined with two dowels, or two dowels with one pin. There is currently no relevant information available about the behaviour of the joint, although practical engineers require such information. A theoretical solution requires simplifications, and these are derived from experimental results. The force method (based on the elastic strain energy) is used for a theoretical solution for a statically indeterminate structure. The results of numerical analysis are very encouraging. The final changes in the stiffness of repaired beams in comparison with the original beams are in very good agreement with experimental results.

1. Introduction

Damaged historical beams loaded by bending moments are usually repaired with scarf joints. The old and new parts of the beam are connected via a scarf joint with pins or with a dowel, or using pins with dowels. An interesting question is, which type of connection is more suitable from the structural perspective. Calculations and experiments that have been performed indicate that the stresses in a scarf joint and the beam stiffness are not reduced by increasing the number of pins or dowels. Practising engineers need simple models and simple instructions for the structural design of joints under the valid EC 5 standard [1]. This standard describes in detail the dimensioning of the elements, and how they are to be connected by steel elements, but traditional connections are not mentioned. Simplifications are needed for a practical model, i.e. we need information on the slip in the cheeks, on the origin of the forces, and on the stiffness of the key. The results and the skills acquired in our study have been used directly in practical applications, e.g. in the medieval Karlštejn Castle (see Fig. 1).

Commercial programmes, such as FEAT, SCIA engineer, FIN, etc., solve the internal forces in roof trusses, beam ceilings and other timber structures. The primary element is a beam subjected to normal stresses parallel to the beam axis (or parallel to the grain, in the case of timber - labelled L), and to the shear stresses acting in the cross sectional plane (labelled T and R) [2]. These stresses exert internal forces, the bending moment M, the axial force N, and the shear force V. The results are used to validate the ultimate bearing capacity of the structure, e.g. pursuant to the EC5 standard. The design provided by the standards works with linear properties of wood and beams. The EC5 standard does not give practical guidance for traditional timber joints, and no theoretical background is available for practical applications. Some information can be found in the literature, e.g. strengthening of traditional joints is described in [3], the design of a twin tenon is presented in [4], a lap joint with dowels is presented in [5,6], and an experimental test of a scarf joint is verified by a calculation in [7–9]. The theoretical design solution of a scarf joint is described in [10,11]. These models show that there is a big influence of the compliance of the assembling pins or dowels on the distribution of forces in the scarf. The required data are obtained from experimental testing and numerical computing.

When timber floor structures are being repaired, it is in most cases the second serviceability limit state that is of decisive

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http://dx.doi.org/10.1016/j.compstruc.2017.03.005
0045-7949/

Fig. 1. Repair to a ceiling beam in Karlštejn castle dowel + 6 pins.

importance. Fulfilling the serviceability state ensures that the structure is in the elastic state.

It is therefore necessary to know the decreasing stiffness of a connected beam, and the role that dowels and pins perform there.

This paper is based on earlier work reported in Fajman, Máca [12]. The current paper includes additional research, in which the stiffness of scarf joints with dowel and two pins was specified by experiments. The obtained experimental results were compared with the theoretical results.

2. Joint

A scarf joint may be a halved joint ($\alpha = 90°$), or a halved joint with oblique cut ends, see Fig. 2. In structural terms, the more favourable option appears to be the halved scarf joint with oblique cut ends with a less steep rake, although the effect of other variables, e.g. the choice of the number of pins (keys, dowels), and their stiffness, on the behaviour of the scarf is not clear [11]. The beam is loaded along its vertical plane. While searching for a solution under certain simplifying conditions, our starting point will be the integral quantities that give us an idea of the behaviour of the joint. In order to determine all forces acting in a joint with precision, it is necessary to supply additional information (or more information under geometrically non-linear behaviour), which can be identified from experiments.

3. Basic relations for joint with two dowels

The load is applied in the vertical direction. Neglecting friction in the vertical plane between the scarves, equilibrium will be sought in integral quantities. This will avoid awkward queries concerning the stress distribution in the scarf cheek. In addition, the derived relations apply both to the linear behaviour and to the non-linear material behaviour of the cheek and the dowel. The stress in a given element may be simplified according to Fig. 3. The scarf cheek is loaded by the axial force and the friction force, while the dowel is loaded by the shear forces and the moment. These forces generate two moments which resist the load.

For the theoretical layout, let us introduce a few simplifications, which will allow us to reduce the structure to a form suitable for 2D analysis:

1. The structure remains within the linear field of deformations and stresses. Fulfilment of the serviceability limit state guarantees that the structure is in the elastic state.
2. The stress is replaced by a resultant with an unknown position.
3. The compressive forces in the scarf cheeks make the bending moment perpendicular to the loading and torsional moment. These values can be neglected when movement out of plane loading is prevented. This condition is met for ceiling beams (held in place by the floor) and for rafters (held in place by the roof boarding). The structure is simplified to a planar structure – the moments which bend the structure perpendicular to the vertical plane have only a small effect on the distribution of forces in the scarf.

The formula will be derived for a scarf with two dowels, and it is evident that the formula for a multi-pin-dowel scarf can be derived by analogy. A theoretical solution of a joint with pins can be found in [10,11]. The forces and their positions are shown in Fig. 3.

The loading of the left part of the beam is represented by the shear force V_f and the bending moment M_f at the beginning of the scarf. Having introduced the condition for friction in the scarf cheek during the displacement of its ends in the downward direction, it holds true for the compressive force $N_\alpha < 0$ that $V_\alpha = N_\alpha\, \mu$, where μ is the friction coefficient for wood. The relations between the oblique and local forces at point 1 can be expressed by the following transformation:

$$N = N_\alpha(\sin\alpha + \mu\cdot\cos\alpha) \quad V = N_\alpha(-\cos\alpha + \mu\cdot\sin\alpha) \tag{1}$$

$$V_1 = N_1\frac{(\cos\alpha + \mu\sin\alpha)}{(-\sin\alpha + \mu\cos\alpha)} = N_1\cdot k_1,$$
$$V_2 = N_2\frac{(-\cos\alpha + \mu\sin\alpha)}{(\sin\alpha + \mu\cos\alpha)} = N_2\cdot k_2 \tag{2}$$

We have three equilibrium conditions – two force conditions in horizontal and vertical directions (3) and the moment condition (4) to dowel 1.

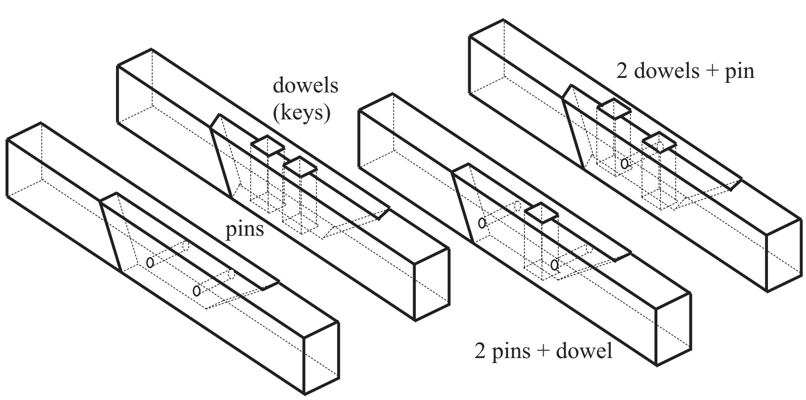

Fig. 2. Scarf joints with pins and dowels.

Please cite this article in press as: Fajman P, Máca J. Stiffness of scarf joints with dowels. Comput Struct (2017), http://dx.doi.org/10.1016/j.compstruc.2017.03.005