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Topology optimization considering multiple loading

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A B S T R A C T

There is an additional new fact that topology optimization has started its career more than hundred years ago by Maxwell and only a few years later by Michell. The classical solutions of the different type of plate or shell problems can be followed by the works of Mroz, Prager and Shield. This paper overviews these almost forgotten results. In addition to the conspectus of this hidden period, the optimal design of curved folded plates is presented. The finite strip method is used for the analysis. At first, a single load case is considered, but later multiple load cases are used for the design. The base formulation is a minimum volume design with displacement constraint, which is represented by the strain energy. For the multiple loading cases two topology optimization algorithms are elaborated: minimization of the maximum strain energy with respect to a given volume and the minimization of the weighted sum of the compliance of the connected load cases with respect to a given volume. The numerical procedures are based on iterative formulae, which are formed by the use of the first order optimality condition of the Lagrangian-functions. The application is illustrated by numerical examples.

1. Introduction

Topology optimization is a very complex computational procedure because it includes the elements of the layout optimization and the optimal cross-section design simultaneously. This paper is an extended, revised version of the conference presentation of the authors [50] and the overview of the early stage of topology design is based on the work of Hemp [20].

Usually the technical papers cite the work of Michell [32] from 1904 as the origin, but the reality is, that the first important work was presented by Maxwell [31]. Maxwell's result was "only" extended by Michell [32] some years later. They presented closed form solutions for the minimum volume structures. They used the word frames but they optimized trusses. More details are presented in the next section.

We also have to mention the works of Kazinczy– the inventor of the plastic hinge theory [23] in 1914- who made significant results in this field but unfortunately these publications have remained hidden. The volume minimization of trusses as an object of the economical design was investigated by him. Kazinczy was also among the first researchers who investigated the problem of the statically indeterminate trusses in case of multiple load conditions [24]. Using the Cremona-type solution procedure, he investigated the case of the pre-stressing technique to reach the uniform collapse of the member forces in the case of statically indeterminate structures. With this technique he used the shakedown theory without naming it, and much earlier than Melan [33] published it in 1936. Kazinczy also discussed the questions of safety and reliability designs much earlier than anybody else in the world.

According to the literature search of the theorem of the optimal design, the topology design remained unnoticed for some forty years until the papers of Foulkes [16], Cox [10–12] and Hemp [20–22], the representatives of the scientific workshop at the Cambrige University. These years around 1955 can be called as the "golden ages" of the optimal layout design of trusses. More than half century ago, several papers were published in this topic. Independently from the English school among others, the papers written by Sved [49] and Barta [6] are one of the most significant ones. The two works have a lot of similarity in both contents and conclusions. In Barta's paper the minimum volume design of plane and space structures (trusses) were discussed. He proved the following theorem: by removing a given number of properly chosen redundant bars from a given network, it is possible to obtain such a statically determinate structure, which yields a structure with the least weight. It has to be noted that this conclusion was stated for single and deterministic loads, as well. Barta also concluded that the proof did not guarantee that only statically determinate structure could be the least weight solution. It is also important to note that the minimum weight designs of different types of structures were studied by Drucker and Shield [13], Mroz [34], Prager and Shield

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[37], Shield [44] and their results are significant to understand the optimality in case of complex problems.

The true blow-up in layout theory was in the 60s and the 70s. In the 60s, the significant publications helped to derive optimality conditions for minimum volume designs. Shield [45] presented optimum design methods for multiple loading. He used variational principles to prove the optimality. In 1960 Schmit [47] applied Full Stress Design (FSD) to statically indeterminate structures and found that FSD provides exact optimum in a single sizing operation for statically determinate structures where the internal forces remain constant during resizing, but for indeterminate structures the number of resizing iterations can vary from a few to many as a function of the sensitivity of internal forces to changes in member sizes. However, Lansing [25] applied FSD to statically indeterminate structures and obtained good results. The reason why these different results were obtained is known now: it mainly depends on the redundancy of structures or in other words, it depends on whether the internal forces remain almost constant during resizing, as it happens in most well designed practical structures. In 1966 Gellatly and Gallagher [18] suggested that FSD should be used to create a ‘‘starting point" of nonlinear programming methods. Furthermore, some important remarks of FSD were reported by Gallagher [17] who pointed out that FSD were inadequate for minimum weight design. Berke and Khot [8] concluded that minimum weight design should be at the point including "fully stressed elements, lower bound elements, and neither of them". During these years Cox [10–12], Hemp [20–22], Prager and co-workers [19,37] elaborated several theories which can be named as the origins of the exact structural topologies. Nagtegaal and Prager [35] investigated the optimal truss layout theory in case of alternative loads. Prager [39] derived an optimality condition for beams and frames subjected to alternating loading by the use of Foulkes mechanism. His results are based on the extension of the optimality conditions presented by Chan [9]. Prager and Rozvany [38] extended the existing optimal layout theory, originally used for low volume fraction, for grid-like structures (trusses, grillages, shell-grids, etc.). The method was validated for the case of not restricting to low volume fraction structures. Rozvany, Olhoff and Bendsoe et al. [41] provided solutions on exact optimal topologies of perforated plates. Achtziger's papers [1–4] present significant achievements in the field of truss designs in the last decades. The three bar truss example was also investigated by Sokol and Lewinski [48].

The most important milestones in deterministic layout and topology optimization should also be reviewed. The origins of the numerical solution technique of the constrained optimality criteria methods (COC) were published by Berke and Khot [8] in 1974. This provided the mathematical background of an effective solution technique in topology optimization. The first numerical procedure for FE (finite element) based topology optimization was elaborated by Rossow and Taylor [40] in 1973, but the real expansion started at the end of 80s [7,42].

Dunning et al. [14,15] introduced an efficient and accurate approach to robust structural topology optimization. The objective is to minimize expected compliance with uncertainty in loading magnitude and applied direction, where uncertainties are assumed to be normally distributed and statistically independent. This approach is analogous to a multiple load case problem where load cases and weights are derived analytically to accurately and efficiently compute the expected compliance and sensitivities.

The plated structures are one of the most frequently used engineering structures. The object of this research work is the optimal design of curved folded plates. There are various solution methods to analyse this type of structures, here the finite strip method is used. At first, a single load case is considered, but later multiple load cases are used for the design. The base formulation is a minimum volume design with displacement constraint which is represented by the compliance. For the multiple loading two equivalent topology optimization algorithms are elaborated: minimization of the maximum strain energy with respect to a given volume or minimization of weighted compliance where the volume of the structure is subjected to volume constraint. The numerical procedures are based on an iterative formula which are formed by the use of the first order optimality condition of the Lagrangian-functions. The application is illustrated by numerical examples.

2. Notes on the layout theory and modelling

The topology design started with the problem class of layout optimization of trusses and the work was called as minimum volume design of frames. As it was indicated earlier, the first optimal solution was elaborated by Maxwell and was later extended by Michell [32] in 1904. It was almost unknown for the optimization community that Michell [32] started his paper by referring to Maxwell's achievements. He determined the optimal layout of a truss for a single load case when the absolute value of the axial stress in any bar is not permitted to exceed a given limit. His solution and the design condition have received lot of attention during the past century, but the optimal layout of a truss for alternative loads seems to get less attention. Somehow, the publications in this topic have remained hidden. Here a brief overview is also performed. It has to be noted that a Michell-truss is statically determinate and the problem class can be handled as one in either elastic design or limit design. The two methods, however, do not any longer lead to the same result when alternative loading or multiple loading cases have to be considered.

The layout theory plays primary importance in structural optimization. The main difficulties are concerned with whether the obtained solution is unique or not, the extremal point is a local or global one. This question is more difficult when several loading cases are taken into consideration. During the 50s–70s of the past century, several papers were published to investigate the questions above. Generally, the variational calculus was the tool to prove the optimality and the uniqueness. Here a limited overview is presented by use of the papers of Nagtegaal and Prager [35] and of Shield [46]. *The overview of these works is based on Hemp [20] presentation in 1958.*

2.1. Maxwell's theorem on minimum volume design

The first important work in truss optimization was presented by Maxwell [31]. He proved a theorem about the equilibrium of a series of attracting repelling centres of force and applied it to trusses (by his original words: to frame structure) in which the bars replaced the action at a distance except in the case of the external forces. Maxwell commented upon scientific significance of his theorem by the use of the following words: ‘‘The importance of the theorem to the engineer arises from the circumstance that the strength of a piece is in general proportional to its section, so that if the strength of each piece is proportional to the stress which it has to bear, its weight will be proportional to the product of stress multiplied by the length of the piece. Hence these sums of products give an estimate of the total quantity of material which must be used in sustaining tension and pressure respectively." We have to notice that Maxwell uses the word ‘‘stress" for what we should term ‘‘load". His result or comment has drawn the practical conclusion about the required weight of the truss.

The Maxwell's problem can be described as follows: consider a truss which maintains equilibrium with a set of forces $\bar{F}_i$ acting at the points $\bar{r}_i$, ($i = 1,2,\ldots,n$). Denote by T_t the load carried in a typical tension member with length L_t and the section area A_t while in