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Entropy solution at concave corners and ridges, and volume boundary layer tangential adaptivity

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ABSTRACT

Two particular aspects of volume boundary layer mesh generation applied to non smooth geometries are considered in this work. First, the treatment of concave ridges and corners is tackled from a generic viewpoint. Entropy satisfying elements are generated where shocks form in the volume. This proves useful to avoid premature halt of the boundary layer, and therefore potential jumps in the normal size. The connection with the Voronoi diagram is commented. Second, boundary layer adaptivity in the tangential plane is considered to honor arbitrary sizing prescription, and avoid size mismatch between the boundary layer and the isotropic sizing.

It is shown that a strict semi-structured framework has to be abandoned in general to accommodate changes in the mesh topology. Size transition between boundary layer and fully unstructured anisotropic mesh is automatically taken into account. Both the concavity problem and the tangential adaptivity are presented together, since they require similar mesh operators. Various numerical examples illustrate the method.

1. Introduction

Boundary layer volume mesh generation applied to generic non smooth surfaces gives rise to various challenges. The mesh should present a strong anisotropy normal to the surface, should smoothly transition to the isotropic region, and should take into account complex ridges and corners. Traditionally [26,29,32,21], the surface mesh is based on an isotropic sizing field. Then, the boundary layer volume mesh is extruded from the surface by generating prisms and by conserving the surface mesh connection. Finally, an unstructured volume mesher fills the gap in the volume. Even though other approaches are available (see [17,1,28] and references therein), it does not alter the following main issues:

- Depending on the surface curvature, stretching and shrinking will be produced, distorting the surface mesh size, forcing premature halt and potential normal size jump.
- Even on flat surfaces, the sizing on the surface may be completely unrelated to the sizing at the extruded location.
- Close boundary layer faces in the three dimensional space may not be aligned and generate anisotropic sizing incompatibilities in the volume.
- There is no guarantee for the exposed front to match the sizing field in general.

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A theoretical tool to take into account the geometry of the surface consists in relying on the Eikonal equation [35], which is a non linear hyperbolic equation capable of generating shocks at concave boundaries, and expansion waves close to convex one. Typically, expansion waves are discretized with multiple normals [10]. However, as opposed to [3], the Eikonal equation is not explicitly computed. It is only used as a theoretical foundation to build the current method.

In the first part of this work, emphasis is given to the reversible phenomenon, where shocks appear. This represents the extension to the three dimensional space of [8]. Only few strategies have been advocated for concave situations. In [26], negative elements are removed, therefore stopping the front progression close to these locations. However, since a small jump between layers is typically enforced, stopping the front prematurely will spread from this location outwards. In [14], the normal direction is limited. However, this may violate the prescribed size, or generate extremely small cells close to concave corners. Another approach consists in using smoothed normals as extruded directions, or even a blend of both [26,29], trading normality for postponed front abortion. It should be pointed out that *smoothing the normal is equivalent to modifying the geometry of the extruded surface*, providing a less accurate top surface. Furthermore, normal smoothing also requires to have enough surface points between discontinuities to be able to smooth these normals, which may not always be the case. Athanasiadis et al. [4] consider special procedures for concave situations, where characteristics coalesce. It is mentioned that a quad surface mesh is expected to be able to collapse the prisms along concave ridges in a structured manner. However, this represents only a particular configuration of a more generic approach presented here. Karman [22] introduces hexes at concave corners and ridges by inserting boundary layers, but no particular detail is given regarding concavities. Ito et al. [18] propose hex generation in concave ridges by remeshing surfaces with quads and extruding at concave ridges and corners. Here also, the generic case is not considered. Chalasani et al. [12] propose quality improvements through generalized elements. They use non-conforming elements to be able to refine and collapse more easily. This is quite close to our approach, but the sizing is guessed during extrusion, and the effect of the surface curvature and the sizing are not clearly highlighted. The Voronoi diagram is not mentioned, while it is the backbone structure of the geometry described in the paper. Kallinderis et al. [20,23] propose to adapt boundary layer meshes by splitting the surface mesh connectivities through all the layers, while the sizing field implies connectivity changes throughout some layers. Shaw et al. [36] also introduce collapsing and enrichment within each layer. However, these modifications are also not conforming, which is not an issue for cell-based finite volume solvers.

In the strategy we propose, the Voronoi diagram [31,33] is the key building block. The Voronoi diagram of some geometrical sites such as points, edges, faces, is a collection of cells such that every point in a cell is closer to its site than to every other site. On surfaces, distances become geodesic distances. With prescribed anisotropy, the Voronoi diagram becomes an anisotropic Voronoi diagram [24]. Considering the boundary layer mesh as a subpart of the three dimensional generalized Voronoi diagram, where triangle faces, edges and vertices are taken into account, the Voronoi bisectors are important spatial locations that ideally would be discretized in the mesh. This would require however the computation of the full three dimensional generalized Voronoi diagram [37,39].

For triangle surface meshes, concavity and convexity are two important notions because they implicitly give information on the distance field generated by those triangles in the three dimensional space. Convexity means that, as seen before, the distance field will present expansion waves, while concavity brings shock waves. Regarding the Voronoi diagram [40], convexity is associated with multiple normals, while concavity implies Voronoi bisectors. As a matter of fact, if the edge shared by two triangles is convex, the extrusion of the two triangles in the normal direction will create a gap in the extruded surface that is filled with a portion of a cylinder. At the opposite, if the edge is concave, an intersection of the triangles will take place. Therefore, concave edges will provide the birth of Voronoi face bisectors while concave corners will generate Voronoi edge bisectors in three dimensions.

The second part of this work is dedicated to the adaptivity of the boundary layer in the tangential plane. Having taken into account the surface geometry in the first part, the boundary layer volume mesh still has to verify the sizing requirements prescribed by the user. The extrusion distance may be large compared to the sizing field variation, potentially creating a discontinuity in it. Furthermore, when marching outwards, the boundary layer mesh may enter an area where the user has prescribed a smaller sizing compared to the surface level. With an anisotropic sizing field that takes into account the sizing implied by the boundary layer, the traditional number of layer parameter becomes meaningless in general, apart from the fact that the user may prefer semi-structured prisms over unstructured tetrahedra in that region. The mesh is sizing compliant in both cases.

This paper is the continuation of the boundary layer mesh generation program proposed in [6]. Similarly to the boundary layer surface generation [8], four main capabilities were identified to generate in a predicted manner a boundary layer volume mesh

- First, an anisotropic sizing field is build that represents the boundary layer anisotropy [5] everywhere in the volume. The sizing field is smooth *in every direction*.
- Multiple normals are generated at convex ridges and corners to take into account expansion waves potentially coupled with shocks in three dimensions [6].
- Concavities are taken into account in order to resolve shocks.
- The boundary layer mesh is adapted in the tangential plane in order to take into account the sizing variations during extrusion.