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Enhanced uncertain structural analysis with time- and spatial-dependent (functional) fuzzy results

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ABSTRACT

Uncertain structural analysis is the computation of uncertain structural response for uncertain input quantities. In terms of structural analysis, the result quantities (e.g. displacements, stresses, damages, ...) are time- and spatial-dependent (τ and $\underline{\theta}$ respectively), such that $x^{u} \mapsto z^{u}(\tau, \underline{\theta})$ needs to be solved.

The usual procedure is to a priori select a small amount of Quantities of Interest (QoI), e.g. for a specific time and spatial location. For this small amount, the uncertainty analysis is performed, but these results do not represent the structural behaviour; they are just an excerpt based on a priori knowledge (in general not available). The consideration of time- and spatial-dependent results yields a high amount of uncertain result quantities.

The main goal of this contribution is the formulation of time and spatial dependency based on the continuous space, which allows the general application to structural analysis. Furthermore, the computation of a large amount of fuzzy result quantities is addressed. Therefore, a fuzzy analysis on the basis of a specific fuzzy sampling is introduced. The third aspect discussed is the enhanced fuzzy structural analysis, an approach to compute, visualise and evaluate functional fuzzy results. The necessity and usefulness of these procedures, in terms of identifying failure modes and weak points, is discussed by two examples.

1. Introduction

Structural analysis and design are a demanding tasks for engineers. The development of an adequate numerical simulation model is one main challenge. The characterisation 'adequate' implies that the simulation model should represent all significant aspects in an optimal way. This engineering task consists of two sub-aspects. First, the *deterministic simulation model* needs to represent the physical behaviour of the observed structure, e.g. the non-linear material behaviour. An engineering approach is to parameterise this model, which means that significant properties need to be identified. In comparison to measurements (also data or information), the parameters of this model are fitted to represent the reality in a best possible manner. A challenge is that usually not 'all' necessary information is available. This is the focus of the second part, the *uncertainty model*. The parameters of the deterministic simulation model are considered as being only partly known. This uncertainty is a major research topic and many ideas exist to represent the uncertainty of data. The overall idea is to find a mathematical formulations, which represents all available information on the structural analysis task without generation of new, not replicable data.

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Notation

ξ	deterministic simulation model
$\square^{\mathrm{f}}$	fuzzy quantity
$\underline{\square}$	vector-valued quantity
x	input quantity
n_{x}	number of input quantities, using $i \in \{1, \ldots, n_{\mathrm{x}}\}$
z	result quantity
n_{z}	number of result quantities, using $j \in \{1, \ldots, n_{\mathrm{z}}\}$
$\underline{k}$	point in the continuous domain
n_{t}	number of continuous dimensions, using $t \in \{1, \ldots, n_{\mathrm{t}}\}$
n_{sim}	number of deterministic simulations, using $l \in \{1, \ldots, n_{\mathrm{sim}}\}$

In general, uncertainty models can be seen as the mapping $\mathcal{U} : Q \to R$. Random numbers can be written as $\mathcal{U}(\Omega, \mathbb{R})$ (with the probability space Ω), while fuzzy quantities can be described as $\mathcal{U}(\mathbb{R}, [0, 1])$. Further characteristics, as the definition of a probability measure P, are not considered in this notation. The methods used in the contribution at hand are non-intrusive and use a deterministic simulation model $\xi : \mathbb{R}^{n_{\mathrm{x}}} \to \mathbb{R}^{n_{\mathrm{z}}} : \underline{x} \mapsto \underline{z}$. This function ξ is the computation of n_{z} result parameters $\underline{z}$ (also outputs or responses) depending on n_{x} input parameters $\underline{x}$ (also features).

Uncertainty has two fundamental properties, *aleatoric* and/or *epistemic* [2]. Aleatoric uncertainty, mainly the natural variability of material properties and environmental conditions, is considered as being stochastic [19]. The epistemic characteristics of uncertainty can be modelled by various approaches, e.g. intervals or fuzzy numbers [8,12]. The combined models lie within the research area 'imprecise probability' and define the foundation of 'polymorphic uncertainty modelling' [5,7,16].

In general, deterministic structural analysis is the computation of time-dependent 3D (spatial) results

$$\xi : \underline{x} \mapsto \underline{z}(\tau, \underline{\theta}). \tag{1}$$

Usually, these results are displacements, strains, stresses, damages or other physically motivated quantities. The time parameter τ and the spatial coordinates $\underline{\theta}$ show that time and spatial dependencies need to be considered for the result quantities $\underline{z}$. To account for the uncertain input and output quantities, the simulation model Eq. (1) is enhanced to give

$$\xi^{\mathrm{u}} : \mathscr{U}(Q,R) \to \mathscr{U}(V,W) : \overbrace{\underline{x}^{\mathrm{u}}}^{\text{prior dependency}} \mapsto \underbrace{\underline{z}^{\mathrm{u}}}_{\text{posterior dependency}} \overbrace{(\tau, \underline{\theta})}^{\text{functional dependency}} \tag{2}$$

The indicator $\square^{\mathrm{u}}$ is used for polymorphic uncertainty modelling and allows a clear mathematical notation independent of the used uncertainty model. In this contribution, the uncertainty model fuzziness is addressed.

In general, for uncertain quantities, four types of dependency can be distinguished. Prior dependency describes the preconditions of the input quantities $\underline{x}^{\mathrm{u}}$, while posterior dependency describes the output quantities $\underline{z}^{\mathrm{u}}$. For fuzzy variables, these dependencies are called input/output interaction. For random variables, the term correlation is established. Furthermore, fuzzy results can be functions with respect to time and space $\underline{z}(\tau, \underline{\theta})$. Fuzzy results depending on one continuous dimension (e.g. fuzzy response processes) are discussed in [9,11,18,21].

These three types of dependencies are indicated by Eq. (2). The fourth dependency is time and space dependency of the uncertain input parameters, e.g. loading processes, spatially varying imperfections or depth dependent soil parameters. For these dependencies, specific approaches as random or fuzzy fields need to be applied [1,4,6].

The main novelty of this contribution is an approach for time- and space-dependent fuzzy results. This method allows the computation of the full fuzzy finite element results, which is in contrast to the common fuzzy structural analysis. Usually, a small amount of representative result parameters is a priori selected, and the fuzzy analysis is performed for these results. These parameters are called 'Quantities of Interest' (QoI). The fuzzy results are only computed for e.g. extreme stresses or extreme displacements. The problem is that no knowledge on the influence of uncertain input parameters on the results is a priori available, therefore the selection of a small amount of QoI is a guess. The contribution at hand gives a method for computing a very high amount of fuzzy result quantities and presents approaches for visualising and evaluating these results.

The contribution is organised as follows. First, the basics of fuzzy analysis are shortly reviewed in Section 2. Second, the organisation of time and spatial dependency of the results is processed by separating the domains of the result quantities and time and space parameters. The used space of continuous parameters is introduced in Section 3. Based on this, the enhanced fuzzy analysis for the structural response is introduced in Section 4. The applicability and the significant improvements of this contribution to uncertain structural design are shown by means of two examples in Section 5. A short overview on the used notation is given at the end.