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Influence of speed fluctuation on cepstrum

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ABSTRACT

This paper deals with the influence of speed fluctuations (non-stationary signal) on the cepstrum. After presenting the cepstrum classic properties, the cepstrum of multiple echoes with a period shift is computed. Examples help to interpret the form of the cepstrum. These shifts could be located in the cepstrum as peaks. Next, a discussion about quefrency resolution is made. It is shown that a low quefrency resolution could produce a fusion of cepstrum peaks (positive and negative).

In order to take account of speed evolution of the signal a time-quefrency representation is proposed. Synthetic signal is used to show how to interpret this representation. Finally, illustrations on time quefrency representation were made by using gear signals by SAFRAN from the Surveillance 8 contest.

1. Introduction

Vibration analysis is a very popular non-intrusive tool to perform diagnostics of rotating machinery. Nevertheless, the effect of rotation speed on the vibration signals became significant when the speed is non-stationary. In this case, instantaneous speed could be measured by using a tachometer signal and used to take account of the speed for diagnostics. The rotating machine instantaneous speed provided by the tacho is also a very useful information. It could be sensitive to mechanical faults [1] and be used for diagnostics. It is also requested for computed order tracking [2].

Since a tachometer speed signal is not always available, instantaneous speed has to be estimated from other signals like the accelerometric signal. Some instantaneous speed extraction methods are described in the MSSP Special Issue on Instantaneous Angular Speed (IAS) Processing and Angular Applications [3,4]. These methods use analytic signal, parametric approach, ...

Another approach based on cepstrum was proposed by the author in the Surveillance 8 Contest [4]. Cepstrum is used as an echo detector [5,6]. The accelerometric signal of the gearbox is considered as a series of echoes of a periodic pattern associated with varying rotation speed. Next, these echoes are measured by a sliding cepstrum (cepstrum of a section of signal extracted using a sliding window) in order to estimate the instantaneous speed. This method is derived from a cepstral order tracking method presented (only in French) in [7].

This method is applied to a signal with non-stationary speed. Unfortunately, the cepstrum is known to be sensitive to speed fluctuations. This sensitivity should be taken into account in order to set up the sliding cepstrum parameters. Therefore, it is proposed in this paper to study the influence of speed fluctuations on the cepstrum.

A first part recalls the classic cepstrum properties and the influence of speed fluctuation. In the second part, a time quefrency-representation is introduced (sliding cepstrum) and speed fluctuation influence is discussed by using some synthetic signals. Next some illustrations are made on real signals.

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2. Cepstrum classic properties

A history of cepstrum analysis is presented in [8,9]. It is shown that cepstrum is applied in many applications including echo detection, diagnosis, modal analysis, spectral harmonic removal, ... This paper uses the cepstrum as an echo detector in order to detect "periodical" repetitive components. This part presents the classical properties of the cepstrum.

Since application is made on a civil aircraft engine gearbox with non-stationary speed, the effect of speed fluctuations on the cepstrum is studied (our contribution).

2.1. Definition

In this paper, the word cepstrum refers to real cepstrum as defined [10] by:

$$cep_{x(t)}(\tau) = \mathcal{F}^{-1}[\ln|X(f)|] \quad (1)$$

where $X(f)$ is the Fourier Transform of the signal $x(t)$, $\mathcal{F}^{-1}$ is the Inverse Fourier Transform, and, ln the naperian logarithm. The variable τ is homogenous with time and is named quefrency.

2.2. Properties

Calculation to explain theoretically the results of the cepstrum on gearbox signals can be found in [6]. Some results of this paper are recalled here.

The accelerometric signal associated with a gearbox can be viewed as a repetition of a series of periodic patterns $p_k(t)$. A periodic pattern at period a_k can be interpreted as a convolution of a single pattern with a Dirac comb of period a_k. Let's consider that the gearbox can be modelled in an area small enough to neglect speed fluctuations as:

$$x(t) = \left[\sum_{k=1}^{M} p_k(t) * em_{a_k}\right] * h_{imp}(t) = x_w(t) * h_{imp}(t) \quad (2)$$

$$em_{a_k} = \sum_{n=0}^{n-1} \delta_{n.a_k} \quad (3)$$

where

- δ_t is the Dirac delta function delayed by t,
- $p_k(t)$ is the pattern associated with the gear k,
- em_{a_k} is multiple echo terms to repeat the pattern at period a_k,
- $h_{imp}(t)$ is the impulse response of the mechanical structure,
- $*$ represents convolution.

The term $p_k(t)$ is considered to have a constant Fourier transform of modulus P_k in [6] (i.e. to have a similar property to white noise). It means that the term $x_w(t)$ could be interpreted as a whitened form of the signal without the contribution of the mechanical structure response.

1. The cepstrum transform a convolution into an addition:

$$cep_{x(t)}(\tau) = cep_{x_w(t)}(\tau) + cep_{h_{imp}(t)}(\tau) \quad (4)$$

It means that the excitation could be separated from the impulse response if their respective cepstrum does not have the same support.

2. Oppenheim and Schafer in [10] studied the impact of transfer function for digitised signal and show that for:

$$H(z) = \frac{B\prod_{i=1}^{M_i}(1-a_i z^{-1})\prod_{i=1}^{M_0}(1-b_i z)}{\prod_{i=1}^{N_i}(1-c_i z^{-1})\prod_{i=1}^{N_0}(1-d_i z)}, \quad (5)$$

the cepstrum is

$$cep_{h(n)}(n) = ln(B), \quad \text{for n} = 0 \quad (6)$$

$$cep_{h(n)}(n) = -\sum_i \frac{a_i^n}{n} + \sum_i \frac{c_i^n}{n}, \quad \text{for n} > 0 \quad (7)$$

$$cep_{h(n)}(n) = \sum_i \frac{b_i^{-n}}{n} - \sum_i \frac{d_i^{-n}}{n}, \quad \text{for n} < 0 \quad (8)$$