ARTICLE IN PRESS

ISA Transactions xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Sliding mode approach for formation control of multi-agent systems with unknown nonlinear interactions

Hossein M.H. Abdoli[a], Majdeddin Najafi[b,*], Iman Izadi[a], Farid Sheikholeslam[a]

[a] Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran
[b] Research Institute for Avionics, Isfahan University of Technology, Isfahan 84156-83111, Iran

ARTICLE INFO

Keywords:
Multi-agent system
Distributed control
Nonlinear interaction
Super twist algorithm

ABSTRACT

This paper proposes nonlinear control approaches to solve a leader-follower formation of multi-agent system with unknown nonlinear interactions. Two distributed sliding mode control approaches are suggested here to track a leader in a desired formation with compensating unknown nonlinear terms. The nonlinear interaction terms can appear in multi-agent systems due to physical connections or cooperation between agents. Also the uncertainty in coefficient of control input is considered. Super twist algorithm is suggested for investigating this problem. Some Lyapunov functions are modified and employed to prove maintaining the formation of group, using the proposed sliding mode controllers. A simulation result for slung load transporting with quad-rotors is presented to demonstrate the capability of the proposed approaches.

1. Introduction

In recent years, multi-agent systems have been the focus of many researchers. This is due to the fact that such systems can show many advantages comparing to single-agent yet complex system. The advantages include lower costs and resources, ability of the system to complete more tasks for instance load carrying, distribution of computational load to multiple resources, etc. [1]. Multi-agent systems have found applications in a variety of topics including microsatellite [2], underwater autonomous vehicles [3], automated highway systems [4], mobile robots [5], and power distribution systems [6].

In multi-agent systems, different problems have been studied including consensus [7], formation [8], flocking [9], containment [10], assigning cooperative tasks [11], etc. In the cooperative control of multi agent systems for a specified target, nonlinear interactions can be introduced in the system. For example in load transportation by a multi agent system, physical connection leads to interconnections between them. These interactions may create uncertain and nonlinear terms in the agents' model. To deal whit these problems, H_∞ [12], adaptive control [13,14], sliding mode control [15] are suggested in recent years. Sliding mode control is more common than other approaches due to the simplicity and efficiency of this approach in theory and applications [16].

Sliding mode offers better performance and less complexity compared to other nonlinear methods [17]. This is achieved by keeping the system contained within some constraints using high frequency switching control signal [18]. Despite many advantages such as robustness and accuracy, the standard sliding mode approach has a few limitations including the infamous chattering [19] and limited relative degree [18]. To cope with chattering a few modifications have been proposed over the years, for example high controller gain and saturation function. One of the proposed methods is the so-called rth order sliding mode control [18], which in addition to solving the chattering, removes the limitations on relative degree. The rth order sliding mode control, requires the knowledge of the $r-1$ derivatives of the sliding surface, which causes computational complexity. However if we use the 2nd order sliding mode control, also known as super twist algorithm, the derivatives of the sliding surface are not required. The general rth order sliding mode control, as well as the special case of super twist have been used to design controllers for nonlinear and multi-agent systems [20–22].

The problem of appearance an uncertainty in multi-agent systems was investigated in many papers, e.g. Refs. [12,14,15,23]. The model of uncertainty may be considered parametric [14,24] or non-parametric [25,26]. However, most of existence researches consider the uncertainty in the states' coefficients and dependent to dynamic of each agent.

This paper considers nonlinear second order multi-agent systems with unknown nonlinear interactions. The nonlinear interactions between neighbor and non-neighbor agents create some uncertainties in

* Corresponding author.
E-mail address: majd_najafi@cc.iut.ac.ir (M. Najafi).

https://doi.org/10.1016/j.isatra.2018.07.029
Received 9 May 2017; Received in revised form 4 April 2018; Accepted 22 July 2018
0019-0578/

states and inputs coefficient. In other words, the dynamics of each agent is affected by the behavior of other agents which includes non-neighbor agents. However, each agent only receives information from its neighbors. The effect of other agents (non-neighbor's) in the group is modelled as non-parametric bounded uncertainty. A sliding and integral sliding mode control are employed to compensate uncertainties and obtaining the goal. The proposed methods need to access different information. Then, stability of the controller is discussed using Lyapunov function. A simulation example of load transportation by a multi agent system is included to demonstrate the effectiveness of the proposed method.

The rest of paper is organized as follows: In section 2, some common definitions in graph theory and Laplacian matrix in multi-agent system are introduced. Also, the model of agents, the problem and uncertainty are defined. In section 3, the process of designing sliding mode and integral sliding mode controllers with Proof of stability are presented. In section 4, a common slung load transportation with multi quad-rotors, is modelled and controlled with proposed methods, to show the capability of these approaches.

2. Preliminaries and problem definition

2.1. Graph theory

Consider a group of n agents (vehicles). The information flow between all agents is described by a graph $G = (\nu, \varepsilon)$ where ν and ε are the nodes and edges sets, respectively. The edge $(\nu_j, \nu_i) \in \varepsilon$ means agent i can access the information of agent j. Each edge $(\nu_j, \nu_i) \in \varepsilon$ has a weight $a_{ij} \geq 0$. The set of the neighbors of agent i is defined as follows.

$$N_i = \{\nu_{ij} | a_{ij} > 0\} \tag{1}$$

If $a_{ij} = a_{ji}$ for all agents, then the graph is called undirected. Agent i is connected to agent j, if there exist a sequential edges from node ν_i to node ν_j. The graph is called connected, if all nodes are connected. The adjacency matrix of a graph is defined as $A \triangleq [a_{ij}] \in \mathbb{R}^{n\times n}$ with elements $a_{ij} > 0$ if $(\nu_j, \nu_i) \in \varepsilon$ and zero otherwise. The graph's Laplacian matrix $L = [l_{ij}]$ is defined as follows.

$$l_{ij} = \begin{cases} -a_{ij} & \text{if} \quad i \neq j \quad \text{and} \quad j \in N_i \\ \sum_{k\in N_i} a_{ik} & \text{if} \quad i = j \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

We also define matrices $\widetilde{L}$, D and B as:

$$\begin{cases} L = D - A \\ D = \operatorname{diag}\left[\sum_{j\in N_i} a_{ij}\right] \\ B \triangleq \operatorname{diag}(b_1, b_2, \ldots, b_n) \end{cases} \tag{3}$$

where coefficients b_i determines which agent has the information of the leader. Leader is an agent with no neighbors and acts autonomously.

Note. in this paper, for every nonzero vector $x \in \mathbb{R}^n$, $|x|$ represents the absolute value of vector's elements and $\frac{x}{|x|^p} \triangleq \left[\frac{x_1}{|x_1|^p}, \ldots, \frac{x_n}{|x_n|^p}\right]^T$. Also, $sig^p(x)$ is defined as $sig^p(x) \triangleq \frac{x}{|x|^p}$.

2.2. Problem definition

Consider the three dimensions dynamical model of an agent with nonlinear interactions with other agent as:

$$\begin{cases} \dot{x}_i = v_{xi} \\ \dot{v}_{xi} = f_{xi}(X, V) + g_{xi}(X, V)u_{xi} \\ \dot{y}_i = v_{yi} \\ \dot{v}_{yi} = f_{yi}(X, V) + g_{yi}(X, V)u_{yi} \\ \dot{z}_i = v_{zi} \\ \dot{v}_{zi} = f_{zi}(X, V) + g_{zi}(X, V)u_{zi} \end{cases} \tag{4}$$

with

$$\begin{cases} X \triangleq [x_1, y_1, z_1, \ldots, x_n, y_n, z_n]^T \in \mathbb{R}^{3n} \\ V \triangleq [v_{x1}, v_{y1}, v_{z1}, \ldots, v_{xn}, v_{yn}, v_{zn}]^T \in \mathbb{R}^{3n} \end{cases} \tag{5}$$

where x_i, y_i and z_i are the coordinates and v_{xi}, v_{yi} and v_{zi} are the velocity of agent i in 3 dimensions. Define the control input vector for the multi-agent system as:

$$\begin{aligned} &U \triangleq [u_i]^T \in \mathbb{R}^{3n} \\ &u_i \triangleq [u_{xi}, u_{yi}, u_{zi}]^T \end{aligned} \tag{6}$$

where u_{xi}, u_{yi}, and u_{zi} are the control input to each dimensions of agent i, $i = 1, 2, \ldots, n$.

$f_i \triangleq [f_{xi}, f_{yi}, f_{zi}]^T \in \mathbb{R}^3$ and $g_i \triangleq \operatorname{diag}[g_{xi}, g_{yi}, g_{zi}] \in \mathbb{R}^{3\times 3}$ include nonlinear interactions and parameters changes. By considering agent dynamic as (4), the general multi-agent dynamics can be presented as:

$$\begin{aligned} &\begin{cases} \dot{X} = V \\ \dot{V} = F(X, V) + G(X, V)U \end{cases} \\ &F(X, V) \triangleq [f_i]^T \in \mathbb{R}^{3n} \\ &G(X, V) \triangleq \operatorname{diag}[g_i] \in \mathbb{R}^{3n\times 3n} \end{aligned} \tag{7}$$

where F and G are vector and diagonal matrix of general uncertainties, respectively.

Assumption 1. *The graph is considered directed and has at least one spanning tree.*

Assumption 2. *The topology of the system is fixed.*

The goal of the multi-agent system is to track the leader in a desired formation, which is defined as:

$$\begin{cases} \dot{x}_L = v_L \\ \dot{v}_L = f_L \end{cases} \tag{8}$$

where $x_L \triangleq [x_l, y_l, z_l]^T$ and $v_L \triangleq [v_{xl}, v_{yl}, v_{zl}]$ are position and velocity of the leader in 3 dimensions, respectively.

According to the target of each agent, the error function is defined for agent i as:

$$\begin{aligned} e_{X_i} &\triangleq \sum_{j\in N_i} a_{ij}(X_i - X_j - \delta_{Xij}) + b_i(X_i - X_L - \delta_{XiL}) \\ e_{V_i} &\triangleq \sum_{j\in N_i} a_{ij}(V_i - V_j) + b_i(V_i - V_L) \end{aligned} \tag{9}$$

where $X_i \triangleq [x_i, y_i, z_i]^T$, $V_i \triangleq [v_{xi}, v_{yi}, v_{zi}]^T$ and $\delta_{Xij} \triangleq [\delta_{xij}, \delta_{yij}, \delta_{zij}]^T$. δ_{Xij} denotes the desired distance of agent i from agent j. By Assumption 1, it is proved that error functions (9) for general multi-agent system are be equivalent to the following error functions:

$$\begin{cases} E_X \triangleq [e_{X_i}]^T = \widetilde{L}[X - \overline{X}_L - \Delta]^T \\ E_V \triangleq [e_{V_{X_i}}]^T = \widetilde{L}[V - \overline{V}_L]^T \end{cases} \tag{10}$$

where $\Delta \triangleq ([1, \ldots, 1]^T)_{n\times 1} \otimes [\delta_{XiL}] \in \mathbb{R}^{3n}$, $\widetilde{L} \triangleq \widetilde{L}_G \otimes I_{3\times 3}$ is modified Laplacian matrix with respect to 3 dimensions problem and e_{X_i}, $e_{V_{X_i}}$, $\widetilde{L}_G$, $\overline{X}_L$, $\overline{V}_L$ are defined as:

$$\begin{cases} e_{X_i} \triangleq [e_{xi}, e_{yi}, e_{zi}]^T \\ e_{V_{X_i}} \triangleq [e_{V_{xi}}, e_{V_{yi}}, e_{V_{zi}}]^T \\ \widetilde{L}_G \triangleq L + B \\ \overline{X}_L \triangleq ([1, \ldots, 1]^T)_{n\times 1} \otimes [x_l, y_l, z_l]^T \\ \overline{V}_L \triangleq ([1, \ldots, 1]^T)_{n\times 1} \otimes [v_{xl}, v_{yl}, v_{zl}]^T \end{cases} \tag{11}$$

Here $\otimes$ denotes the Kronecker product.

By differentiating of general error function (10) and considering Assumption 2, the error dynamics of interconnection graph can be expressed as: