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Research article

A novel approach for benchmarking and assessing the performance of state estimators

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ABSTRACT

State estimation is a widely adopted soft sensing technique that incorporates predictions from an accurate model of the process and measurements to provide reliable estimates of unmeasured variables. The reliability of such estimators is threatened by measurement related challenges and model inaccuracies. In this article, a method for benchmarking of state estimation techniques is proposed. This method can be used to quantify the performance and hence reliability of an estimator. The Hurst exponents of *a posteriori* filtering errors are analyzed to characterize a benchmark (minimum mean squared error) estimator, similar to the minimum variance control benchmark developed for control loops. A distance metric is then used to quantify the extent of deviation of an estimator from the benchmark. The proposed technique is developed for linear systems and extended to nonlinear systems with single as well as multiple measurable variables. Simulation studies are carried out with Kalman based as well as Monte Carlo based estimators whose computational details are significantly different. Results reveal that the technique serves as a tool that can quantify the performance and assess the reliability of a state estimator. The strengths and limitations of the proposed technique are discussed with guidelines on applications and deployment of the technique in a real life system.

1. Introduction

The state space form of system description expresses measurements y_k obtained from a system Ξ as functions of external inputs u_k and internal variables x_k of the system as:

$$\Xi: \begin{array}{ll} x_{k+1} & = f(x_k, u_k) + \nu_{k+1} \\ y_k & = h(x_k, u_k) + \omega_k \end{array} \qquad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$. The process uncertainty and measurement noise (denoted ν_k and ω_k respectively) are typically assumed to be normally distributed white noise with zero mean and variance-covariance matrices Q and R respectively. The internal variables (x_k) called state variables often represent physical quantities that cannot be measured or are prohibitively expensive to measure, while at the same time whose knowledge can be valuable in assessing the state of the system (as healthy or faulty, steady or transient, etc.), in monitoring the system and in deciding control actions. At times, these variables provide information that is more critical about the state of the system than can be obtained from measurements alone. For example, in a reactor with gaseous reactants and products the partial pressure of gases (that cannot be measured directly) are more informative about the system's state (say, extent of reaction) than the measurable total pressure; in an aircraft, the slosh of liquid fuel inside the tank is more critical for maintaining balance of the aircraft [1] than the mass of the tank that can be measured. Accurate knowledge of state variables is therefore critical to knowing the current state of operation and margins to operating limits of a system, deciding necessary control actions to maintain the plant and hence for overall system safety. State estimation is a widely adopted tool that incorporates predictions from an accurate model and measurements from the process to provide accurate estimates of state variables of the system.

Kalman [2] developed a recursive prediction-correction based state estimation algorithm which serves as the best unbiased estimator for linear systems with Gaussian noise. The Kalman filter uses a model to predict the states and measurements. It then appropriately weighs the predicted state variables and the plant measurements to minimize the corrected state error covariance and provides the corrected states. This predictor-corrector algorithm of the Kalman filter has been shown to be best linear unbiased estimator (BLUE). These properties of the Kalman filter can be attributed to the assumption of all random variables being

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Gaussian along with the linearity of the system. This results in the Gaussian nature of state estimates being preserved over all time and the filter being optimal. Owing to nonlinear operations performed on the variables resulting in non-Gaussian nature of the state estimates, a best optimal estimator (in a mean squared error sense) does not exist for nonlinear systems. However, extensive research has led to a host of estimators being developed for these systems that perform different approximations to the estimation problem.

The Kalman filter has been extended to non-linear systems resulting in the extended Kalman filter, which approximates the non-linearity by using a Taylor series approximation so that the Kalman filter equations can be used for state estimation. Variants of the Kalman filter (particle filters [3,4] and ensemble Kalman filter [5–7]) have appeared in the literature that provide reliable state estimates for more challenging and non-linear systems. These estimators approach the state estimation from different, albeit based on similar fundamental concepts, vantage points. For instance, the particle filter is a Monte Carlo estimation method that performs a random sampling of states (particles) to propagate through the model, and uses a likelihood function to obtain corrected state estimates. However, the extended Kalman filter is the most popular filtering technique for state estimation of non-linear systems and has been applied in several applications such as global positioning system, aircraft inertial navigation system etc. The Unscented Kalman filter (UKF) [8,9], developed in the early 2000s addressed the non-linear state estimation problem as one of approximating probability density function of state estimates under a non-linear transformation rather than approximating the non-linearity itself and was shown to outperform EKF with comparable computational expense (for small values of n). The cubature Kalman filter (CKF) [10] is also similar to the UKF which has been proposed as an alternate method of nonlinear estimation. The EKF and UKF have been modified to handle differential algebraic equation (DAE) systems [11] and systems with constraints on state variables [12,13]. A moving horizon approach to handle constraints for linear state estimation was proposed in Ref. [14], which was then extended to non-linear systems [15] with the idea of capturing more information with a window of past estimates as compared to the recursive EKF and UKF. Although this technique does capture more information than the recursive extension of Kalman filter, its large computational expense resulted in research attention being focused on developing recursive estimators. Recently, a recursive state estimator (receding horizon non-linear Kalman filter) [16] was proposed for non-linear systems which was shown to perform as good as MHE at a much lower computational expense.

1.1. Motivation

Despite the diversity and sophistication of state estimators, they inevitably require two pieces of information to provide reliable estimates of the state variables - the process model (Equation (1)) and measurements y_k. This renders model based estimators vulnerable to measurement issues (such as delayed measurements, missing data) and model plant mismatch (i.e., disagreement between process dynamics and model description). Measurement related challenges have been addressed in the literature [17–19] that have resulted in modified versions of the standard estimation (EKF and UKF) algorithms.

On the other hand, model plant mismatch can occur in the form of parametric deviations of the model (f, h, Q and R) or errors in the assumed structure of the model (e.g. measurement and state equations corrupted with coloured noise instead of IID process). Such mismatches can be due to changes in plant dynamics over time as a result of change in operating conditions or wear and tear of the process equipments. When there is a mismatch between the model and plant dynamics, there is a conflict between the two pieces of information fed to the estimator, i.e., the process model (predictions) and measurements. This may result in biased or divergent state estimates, which can misguide a plant operator. Furthermore, for nonlinear systems, different estimators adopt different approaches and make different approximations to the estimation problem, that can have a varied impact on the performance and sensitivity to MPM.

The literature has widely acknowledged MPM as a threat to the reliability of estimators. Techniques have been suggested to *avoid* MPM by cautious selection of parameters and by suitably incorporating unmodelled effects in the noise covariance matrices [20]. The innovation sequence test has been proposed to detect MPM [21]. Error bounds for the state estimates in the presence of non-Gaussian noise corrupting the system have been provided using probability theory in Refs. [22–25]. An alternative version of the Extended Kalman filter has also been recently proposed to address modeling uncertainties in Ref. [26].

However, quantification of the impact of MPM on an estimator's performance has not been extensively addressed. The most straightforward approach is to use the mean squared error (MSE) of filtered measurements as an index of performance. However, this approach does not allow one to benchmark the estimator. In other words, the ideal value of MSE for a system is not known beforehand. As a result, one can only detect an increase or decrease in the MSE, thereby preventing MSE from being used as an absolute measure of performance.

1.2. Contributions

As discussed earlier, the performance of an estimator depends on the quality of the model (extent of MPM) and the severity of approximations made by the (nonlinear) estimator. In this article, we attempt to assess an estimator with respect to a benchmark filter in the presence of both factor affecting performance. However, since the approximations made by any particular estimator are the same, the present study reduces to quantifying the performance of the estimator with varying levels of model plant mismatch. We therefore refer to this problem as quantifying the impact of MPM on a state estimator and discuss the influence of the approximations made by the estimator in a separate section. Specifically, we (i) develop method to characterize a benchmark estimator such as the Kalman filter with measures other than MSE and (ii) quantify the performance of an estimator in the presence of MPM for systems with multiple measurable variables.

We first characterize the Kalman filter which is the optimal estimator for linear systems in Section 2. These properties are used to derive the conditions for individual performance metrics of the ideal multivariate estimator (without MPM and approximations) in Section 3, which are then used to devise a performance measure of any estimator. Simulation results are presented in Section 4 followed by discussion in Section 5 and concluding remarks and directions for future work in Section 6.

2. Preliminaries

Consider a system described by Equation (1). The problem of estimation involves providing reliable estimates of state variables (x_k) from measurements (y_k) in the presence of model uncertainty (ν_k) and noise in measurements (ω_k). Here we address estimation in the context of filtering, i.e., obtaining state estimates at time k given measurements till time k, i.e., $\hat{x}_{k|k}$ as opposed to prediction and smoothing that use past and future measurements respectively to estimate state variables.

2.1. Innovation properties of the Kalman filter

The Kalman filter, which is the best linear unbiased estimator (BLUE) solves the above estimation problem in two steps: (i) prediction, in which the state estimates and their covariances are propagated through the model and (ii) correction, in which a weighted combination of the predicted and observed measurements is used to correct the state estimates and covariances thereof. For a linear system, Equation (1) can be written as: