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Research article

Fast Kalman-like optimal FIR filter for time-variant systems with improved robustness☆

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ABSTRACT

In this paper, a fast Kalman-like iterative OFIR algorithm is proposed for discrete-time filtering of linear time-varying dynamic systems. The batch OFIR filter is re-derived in an alternative way to show that this filter is unique for such systems. A computationally efficient fast iterative form is found for the OFIR filter using recursions. It is shown that each recursion has the Kalman filter (KF) predictor/corrector format with initial conditions specified via measurements on a horizon of N nearest past points. In this regard, the KF is considered as a special case of the iterative OFIR filtering algorithm when N goes to infinity. Applications are given for the 3-state target tracking and three-degree-of-freedom (DOF) hover system. It has been shown experimentally that the proposed iterative OFIR algorithm operates much faster than the batch OFIR filter and has the computational complexity acceptable for real-time applications. It has also been demonstrated by simulations that an increase in the number of the states results in better robustness of the OFIR filter against temporary model uncertainties and in higher immunity against errors in the noise statistics.

1. Introduction

Information gathering about physical processes and dynamic system states plays a key role in diverse branches of science and engineering. Requirements of high accuracy of state estimation often go along with the necessity of providing decisions in real time when even small delays are not tolerated. Examples can be found in measurements [1], automation [2], navigation systems [3], mobile robotics [4], control [5], and telecommunications [6]. In such cases, optimal real-time estimators, called filters, are required. The Kalman filter (KF) [7–10] is the most widely used real-time optimal estimator. However, the KF is a Bayesian estimator and its recursive algorithm has the infinite impulse response (IIR), owing to which the KF often suffers of insufficient robustness [11]. Better robustness is inherent to finite memory filters [12–15] and to filters with finite impulse response (FIR) [16–18].

Unlike the KF, the FIR filter utilizes measurements on an interval of N most recent neighbouring points called *horizon*. Compared to the KF, FIR filters demonstrate many useful properties such as the bound input/bound output (BIBO) stability [11], higher robustness against temporary model uncertainties [12,19] and round-off errors [16], and lower sensitivity to noise [20]. The most noticeable early works on optimal FIR filtering are [11,12,21]. At that time, the analytical complexity and large computational burden caused difficulties in using FIR filters for state estimation. Nowadays, the interest to FIR filtering grows due to the tremendous progress in the computational resources and some analytical innovations. Accordingly, one finds a number of new solutions on FIR filtering [22–29], smoothing [31,32], prediction [20,33], and efficient applications [34–37].

For example, the receding horizon iterative Kalman-FIR filter was derived for time-invariant systems in Ref. [22] from the formulation of KF, and it has been shown that this algorithm possesses the unbiasedness and deadbeat properties irrespective of the initial values. For time-variant systems, a finite-horizon KF was proposed in Ref. [38]. A receding horizon state observer was derived within the framework of least squares [23]. A minimum variance unbiased FIR filter was proposed in Ref. [24] in both the batch and fast iterative forms. For the same model as in Ref. [24], the fixed-lag minimum variance FIR smoother was developed in Refs. [30,31], where the target solution was achieved by minimizing the mean square errors (MSEs) constrained by the unbiasedness condition. In Ref. [32], a low complexity Kalman-FIR smoother was derived under the assumption that the state transition matrix is invertible. For real-time invariant state-space model [39], the p-shift batch optimal FIR (OFIR) estimator was proposed in Ref. [33] and further extended in Ref. [40] to time-variant systems. A novel fast

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iterative unbiased FIR (UFIR) filter was derived for linear systems in Refs. [25,33] and extended to nonlinear models in Ref. [26]. Quite recently, a batch minimum variance UFIR filter was developed in Ref. [41] and a fast iterative OFIR algorithm proposed in Ref. [42] for time-invariant systems. Although fast estimation is a common issue in real-time applications, fast Kalman-like iterative OFIR filtering still has been developed only for time-invariant systems [42]. A more general and demanded solution for time-variant systems has not been addressed so far. This essentially limits dissemination of OFIR filtering in engineering practice and motivates our present work.

In this paper, a fast iterative OFIR algorithm is proposed for a more general case of time-variant systems. We first provide an alternative derivation of the batch OFIR filter to show that this filter is unique for linear discrete-time state-space models. We then find a fast iterative form for this filter and show that each iteration is the Kalman-like recursion with the special initial conditions and bias correction gain. Fast computation of the MSE is also provided for the OFIR filter. We finally compare the OFIR filter to KF based on the 3-state switching Markov system and 3-degree-of-freedom (DOF) hover system. The remainder of this paper is organized as follows. In Section 2, we give preliminaries and formulate the problem. The batch OFIR filter is considered in Section 3. The iterative form of the OFIR filter is derived in Section 4 and estimation errors are discussed in Section 5. Applications to target tracking and hover system of a flying apparatus is given in Section 6. Finally, conclusions can be found in Section 7.

The following notations are used: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $E\{\cdot\}$ denotes the statistical averaging; diag $(e_1 \cdots e_m)$ represents a diagonal matrix with diagonal elements $e_1, \ldots, e_m$; tr(M) is the trace of M; and I is the identity matrix of proper dimensions.

2. Preliminaries and problem formulation

Consider a general class of discrete-time linear systems represented in state-space with time-variant coefficients as

$$x_k = A_k x_{k-1} + B_k w_k \tag{1}$$

$$y_k = C_k x_k + v_k \tag{2}$$

in which k is the discrete time index, $x_k \in \mathbb{R}^n$ is the state vector, $y_k \in \mathbb{R}^p$ is the measurement vector, and $A_k \in \mathbb{R}^{n\times n}$, $B_k \in \mathbb{R}^{n\times u}$, and $C_k \in \mathbb{R}^{p\times n}$ are time-variant matrices. Here, $w_k \in \mathbb{R}^u$ and $v_k \in \mathbb{R}^p$ are additive process and measurement noise sources with known covariances $Q_k = E\{w_k w_k^T\}$ and $R_k = E\{v_k v_k^T\}$, respectively. We suppose that w_k and v_k are zero mean, white, and mutually uncorrelated; that is, $E\{w_k\} = 0$, $E\{v_k\} = 0$, $E\{w_k w_j^T\} = 0$ and $E\{v_k v_j^T\} = 0$ for all k and $j \neq k$, and $E\{w_k v_j^T\} = 0$ for all k and j.

The FIR filter requires simultaneously N data points taken from the horizon $[l = k - N + 1, k]$. Therefore, (1) and (2) need to be extended on $[l, k]$. That can be done if to use the recursively computed forward-in-time solutions [40] and write

$$X_{k,l} = A_{k,l} x_l + B_{k,l} W_{k,l} \tag{3}$$

$$Y_{k,l} = C_{k,l} x_l + H_{k,l} W_{k,l} + V_{k,l} \tag{4}$$

where the extended vectors are $X_{k,l} = [x_k^T, x_{k-1}^T, \ldots, x_l^T]^T \in \mathbb{R}^{Nn\times 1}$, $Y_{k,l} = [y_k^T, y_{k-1}^T, \ldots, y_l^T]^T \in \mathbb{R}^{Np\times 1}$, $W_{k,l} = [w_k^T, w_{k-1}^T, \ldots, w_l^T]^T \in \mathbb{R}^{Nu\times 1}$, and $V_{k,l} = [v_k^T, v_{k-1}^T, \ldots, v_l^T]^T \in \mathbb{R}^{Np\times 1}$. The extended k- and N-variant matrices $A_{k,l} \in \mathbb{R}^{Nn\times n}$, $B_{k,l} \in \mathbb{R}^{Nn\times Nu}$, $C_{k,l} \in \mathbb{R}^{Np\times n}$, and $H_{k,l} \in \mathbb{R}^{Np\times Nu}$ can be represented as, respectively,

$$A_{k,l} = [\mathscr{A}_{k,l+1}^T, \mathscr{A}_{k-1,l+1}^T, \ldots, \mathscr{A}_{l+1,l+1}^T, I]^T, \tag{5}$$

$$B_{k,l} = \begin{bmatrix} B_k & \mathscr{A}_{k,k} B_{k-1} & \cdots & \mathbf{A}_{k,l+2} B_{l+1} & \mathscr{A}_{k,l+1} B_l \\ 0 & B_{k-1} & \cdots & \mathscr{A}_{k-1,l+2} B_{l+1} & \mathscr{A}_{k-1,l+1} B_l \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & B_{l+1} & \mathscr{A}_{l+1,l+1} B_l \\ 0 & 0 & \cdots & 0 & B_l \end{bmatrix}, \tag{6}$$

$$C_{k,l} = \bar{C}_{k,l} A_{k,l} \tag{7}$$

$$H_{k,l} = \bar{C}_{k,l} B_{k,l} \tag{8}$$

where

$$\begin{aligned} \bar{C}_{k,l} &= \text{diag}(C_k C_{k-1} \cdots C_l) \\ \mathbf{A}_{i,j} &= \textstyle\prod_{r=0}^{i-j} A_{i-r} = A_i A_{i-1} \ldots A_j\,. \end{aligned} \tag{9}$$

At the initial horizon point, (3) becomes $x_l = x_l + B_l w_l$ that is uniquely satisfied if w_l is zero-valued, provided that B_l is not zeroth. That means that the initial state must be known in advance or estimated optimally. For more detail about this model see Refs. [25,43].

The FIR filtering estimate can be obtained at k via (4) using the discrete convolution as

$$\hat{x}_{k|k} = K_k Y_{k,l} \tag{10}$$

where $\hat{x}_{t|r}$ means the estimate at t via measurements from the past to and including at r and K_k is the FIR filter gain, which needs to be defined to obey some cost function. Note that the aforementioned inherent properties of FIR filtering are associated with the fact that measurements prior to l are discarded in (10) and thus do not affect the estimate [25,27,41], unlike in the KF which has IIR. It is also necessary to emphasize that when the system considered is time-invariant, the FIR estimate (10) will becomes $\hat{x}_{k|k} = K_N Y_{k,l}$, which means that the filter gain K_N is time-invariant and can be determined off-line once the horizon length N is available. In this case, K_N is not necessarily to be realized into iterative computation structure, although corresponding results have been shown in Ref. [42] to provide an insight into the OFIR filter.

The optimal gain K_k can be obtained for (10) in the minimum MSE sense by minimizing the trace of the MSE as

$$\hat{K}_k = \arg\min_{K_k} E\{\text{tr}(e_k e_k^T)\} \tag{11}$$

where $e_k = x_k - \hat{x}_{k|k}$ is the estimation error. Provided $\hat{x}_{k|k}$ via (10), the one-step prediction required by feedback control and associated with receding horizon filtering [16] can be formed as $\hat{x}_{k+1|k} = A_{k+1}\hat{x}_{k|k}$, similarly to the KF.

The problem now formulates as follows. Given the model, (1) and (2), we first wish to make sure that the OFIR filter [40] is unique for linear models by deriving it in an alternative way. We then would like to find a fast iterative form for this filter using recursions and connect it to the KF. Finally, we want to test the OFIR filter by a 3-state target tracking system and 3-DOF hover system of a flying apparatus to investigate the trade-off with the KF.

3. Batch OFIR filter

The batch OFIR filter was originally derived in Ref. [40] by employing the orthogonality condition. To show that this filter is unique for (1) and (2), in this section we provide its alternative derivation. To this end, the following lemma will be used.

Lemma 1. *The trace optimization problem is given by*

$$\begin{aligned} &\arg\min_K \text{tr}[(KF - G)H(KF - G)^T \\ &+ (KL - M)P(KL - M)^T + KSK^T] \end{aligned} \tag{12}$$

where $H = H^T > 0$, $P = P^T > 0$, $S = S^T > 0$, and F, G, H, L, M, P and S are constant matrices of appropriate dimensions. A solution to (12) is given