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Research article

A novel weighting method for multi-linear MPC control of Hammerstein systems based on included angle

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ABSTRACT

A novel included angle based weighting method is proposed for multi-linear model predictive control (MPC) of Hammerstein systems. It makes full use of the special structure of the Hammerstein models, and thus it is intuitive and simple. Moreover, there is only one tuning parameter and the weights can be calculated offline and stored in a look-up table. Therefore, online computational load is largely reduced. Most important of all, it schedules local controllers properly and effectively. A Lab-tank system which can be modeled into a Hammerstein model is investigated. Comparisons are made among the nonlinearity inversion control method, the proposed weighting method and traditional weighting methods, e.g., Trapezoidal and Gaussian weighting methods. Simulations confirm that the proposed weighting method is superior to traditional methods.

1. Introduction

The Hammerstein modeling structure has been a popular nonlinear model for it can account for the nonlinear characteristics of many industrial processes [1–9]. Generally, the control method for Hammerstein systems is the nonlinearity inversion control method [4,5]: Firstly, a linear controller is designed based on the linear part of the system; secondly, the linear controller is converted into a nonlinear one by the inversion of the static nonlinear part. Nevertheless, the nonlinearity inversion method fails when there exists input multiplicity. In addition, the nonlinearity inversion method may not cover the influence of the input nonlinearity and may easily degrade the closed-loop performance [5].

Recently, the multi-model control method [10–22], especially the multi-linear MPC method has been employed to solve the control problem of Hammerstein systems [10–12]. The key to the multi-linear MPC control approach is using a set of linear models to approximate the nonlinear system, designing a linear MPC controller for each linear model, and finally combining the linear MPC controllers into a global control, i.e., the multi-linear MPC controller [13]. There are various methods for multi-linear MPC combination. Generally, there are two types of combination, switching and weighting [13]. The switching methods may lead to output oscillation, while the weighting methods make the output smooth [14]. Therefore, the weighting methods are adopted and discussed in this work.

There are various kinds of weighting methods in multi-model control of nonlinear systems, such as, the Trapezoidal functions [15,22], the Gaussian functions [16], the recursive Bayes weighting method [18,21], the output error method [13], and so on. However, the most popular weighting methods are still the traditional Trapezoidal and Gaussian methods [22], mainly because they are simple and easy to implement. Besides, their weights are independent of time instant and thus can be calculated offline and stored in look-up tables, which reduces the online computational load. Unfortunately, there are many tuning parameters in the traditional methods, which make the tuning process rather complicated and troublesome. Methods, such as the recursive Bayes weighting method and the output error method, need the errors or relative errors at every time instant [21] and thus have to be computed online, which involves large computational load.

In order to take the advantages of the special structure of the Hammerstein model, and continue the merits of the traditional weighting methods but avoid their drawbacks, the multi-linear MPC approach is introduced into the Hammerstein systems, and the included angle is employed to formulate a novel and effective weighting method in this work. The included angle is defined by the difference of slope angles at two different static points along the static nonlinear function of the Hammerstein system. Since the nonlinearity of a Hammerstein system lies in its static element, the included angle which reflects the variation of slope can be used to measure the nonlinearity of the Hammerstein system. And further, we can use the included angle to define weighting functions which can schedule the local MPCs according to the nonlinearity of the Hammerstein system. Besides, in

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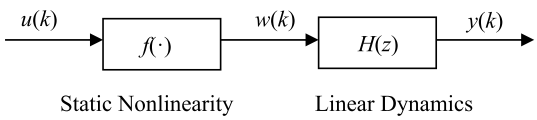

Fig. 1. The structure of a Hammerstein model.

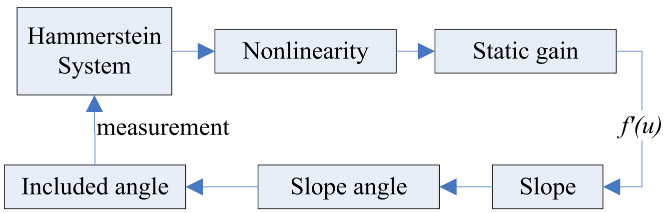

Fig. 2. The relationship between included angle and nonlinearity.

Table 1
Operating points of the linear model bank for the Lab-tank.

Subregion	1st	2nd
Operating point (x,u)	GP_{10}: ([38.8125, 18.2493], 38.8125)	GP_{26}: ([43.3125, 48.9460], 43.3125)

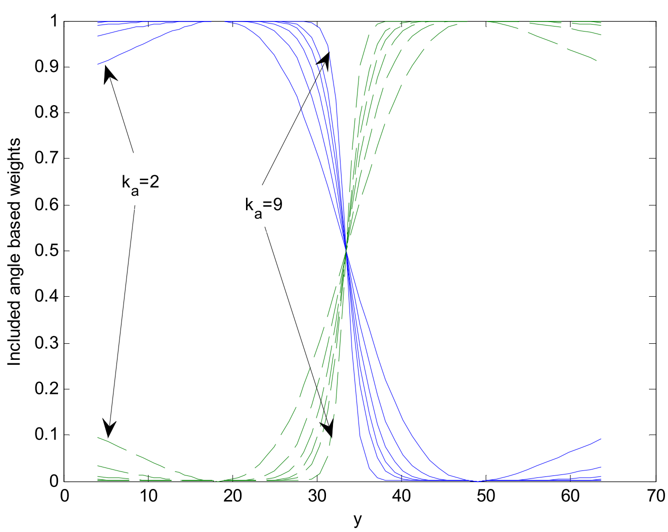

Fig. 3. Weights based on included angle for the Lab-tank system.

order to make the closed-loop tuning process simple and easy, only one tuning parameter is introduced in the proposed included angle weighting method. A Lab-tank system which can be approximated by a Hammerstein model is studied. Comparisons have been made between the included angle based weighting method and the traditional Trapezoidal and Gaussian weighting methods. Simulations demonstrate the effectiveness of the proposed weighting method.

The arrangement of the paper is as follows. In section 2, the special structure of the Hammerstein model is analyzed. In section 3, the included angle based weighting method is proposed. Multi-linear MPC for Hammerstein systems is detailed in Section 4. Comparisons and discussions are made in Section 5. Section 6 concludes the paper.

2. Analysis of a Hammerstein model

As shown in Fig. 1, a Hammerstein model is comprised by a connection of a static nonlinear function $f(\cdot)$ and a linear dynamical system $H(z)$, and $f(\cdot)$ is in front of $H(z)$ [1,2]. If the two parts are in an inverse order, a Wiener model is formed. The mathematical model of a Hammerstein system in the discrete time domain is shown in Eq. (1).

$$\begin{cases} w(k) = f(u(k)) \\ y(k) = \sum_{i=1}^{p} a_i y(k-i) + \sum_{i=1}^{q} b_i w(k-i) \end{cases} \tag{1}$$

where u is the input of the system; w is an intermediate variable; y is the output of the Hammerstein system. And $f(\cdot)$ is a nonlinear function, $H(\cdot)$ is a z-transfer function. In this article, we focus on Hammerstein systems that can be represented or approximated by Hammerstein models.

As well-known, the nonlinearity of a Hammerstein system exists in the static gain but not the dynamics. Therefore, the Hammerstein model is thought to be a generalization of the gain-scheduling models for nonlinear systems [3]. That is to say, gained scheduling models, a set of local linear models—the multi-models, can be used to approximate a Hammerstein system. As a popular control method for nonlinear systems, the multi-linear MPC approach, integrating the merits of multi-model control methods and MPC method, can overcome the disadvantages of the dominate control method for Hammerstein systems, e.g., the nonlinearity inversion method. In order to apply the multi-linear MPC control approach to the Hammerstein system, we analyze the properties of the Hammerstein systems, and make full use of them to simplify the design of a multi-linear MPC controller.

For convenience of analysis and without of loss of generality, suppose $\frac{b_1+b_2+\ldots+b_q}{1-a_1-a_2-a_p} = 1$ in Eq. (1), then we can get that the static gain of the Hammerstein system (1) at u_0 is equal to $f'(u_0)$, which was detailed in Ref. [11]. $f'(u_0)$ is the slope of the static input-output (I/O) map, also the static gain of the Hammerstein system (1). Therefore, the variation of static gain of Hammerstein system Eq. (1) is captured by the slope $f'(u_0)$. In other words, $f'(u_0)$ can be used to measure the variation of static gain, *i.e.*, the nonlinearity degree of system (1). However, the variation range of slope is $[-\infty, +\infty]$ [6], making it improper to be a measuring tool. Fortunately, the slope angle has a finite range $[0, \pi]$. Further, the difference between two slope angles is included angle, which reflects the variation of slope angle. Hence, the included angle is used to measure the nonlinearity of a Hammerstein system [17]. The relationship among nonlinearity, static gain, slope, slope angle, and included angle is shown in Fig. 2.

The included angle is defined as [6]:

$$\theta(G_i, G_j): =\theta_{ij} = |\theta_i - \theta_j|, \quad i, j = 1, 2, \ldots, n_g \tag{2}$$

where i, j are two steady state points on the I/O map of the SISO Hammerstein system(1); n_g is the number of steady state point; θ_i is the slope angle of the Hammerstein system (1) at steady state point i and $\theta_i = arctan(f'(u_i))$. G_i is the linearized model around the ith steady state point, and θ_{ij} is short for $\theta(G_i, G_j)$. Obviously, the variation range of the included angle is finite: $[0, \pi]$.

If the included angle between two points is small, then the Hammerstein system (1) has a similar characteristic at the two points, and their static gains are close. Otherwise, the Hammerstein system (1) has quite different behaviors at the two points when their included angle is big, usually bigger than $\pi/2$. Based on this feature, a grid algorithm is proposed based on included angle [10] and a division algorithm [12] is developed for decomposing nonlinear Hammerstein systems into linear subsystems.

In this work, we are to take advantage of the special property of Hammerstein systems to develop a novel included angle based weighting method for multi-linear MPC control of Hammerstein systems, namely, combining local MPC controllers into a global one. Just as traditional weighting methods, the definition of the proposed method is simple and intuitive. In contrast to traditional methods, complex tuning task can be avoided, since only one tuning parameter will be introduced into the proposed method. The proposed method will be detailed in the following section.

3. A novel included-angle-based weighting method

For the Hammerstein system (1), firstly, the whole operating range