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Research article

Active disturbance rejection boundary control of Timoshenko beam with tip mass

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ABSTRACT

In this paper, the active disturbance rejection control (ADRC) is utilized to stabilize the vibration of perturbed Timoshenko beam model with tip mass. The boundary control design is based on a hybrid PDE–ODE model, and is accompanied with designing a high-gain extended state observer (ESO) that is used to estimate the boundary disturbances. By transforming the model into the appropriate state space, the semigroup theory is employed to prove the well-posedness of the closed-loop system. To this end, it is proved by a frequency domain method that the semigroup generated by the system operator is exponentially stable, which allows to conclude the boundedness of perturbed closed-loop system response. The stability of the closed-loop system is further analyzed using the Lyapunov approach. Simulation results are presented to illustrate the efficacy of the suggested method.

1. Introduction

Flexible structures are the inseparable part of many important industries due to increasing demand for economical and energy-efficient mechanical systems. By oscillatory behavior of flexible members in response to external excitations and the challenges lying ahead of suppressing their vibration and its consequences, the control of flexible structures has emerged as a thriving research area that has attracted the researchers during recent decades [1,2]. When represented mathematically, the dynamics of mechanical systems including rigid and flexible parts is described by a set of coupled ordinary differential equations (ODEs) and partial differential equations (PDEs) [3]. The mathematical challenges in designing high-performance control laws for such hybrid PDE-ODE models have generated considerable trend towards development of new control tools [4,5].

In studying the vibration of hybrid mechanical systems, the use of beam element is widespread in modelling the flexible parts. The most commonly used beam model is based on the Euler-Bernoulli theory that provides a good description of thin beams, i.e. beams with small thickness to length ratios [6]. For thicker beams, a more accurate model is obtained using the Timoshenko theory that accounts for rotary inertial energy and the beam deformation due to shear. However, since the Timoshenko model is a more difficult model due to its higher order, the control design for flexible structures has been mainly based on the Euler-Bernoulli theory [7].

Studies on exponential stabilization of cantilevered Euler-Bernoulli beams can be found in Refs. [8–11]. The control of Euler-Bernoulli beam with boundary disturbances is investigated in Refs. [12–14]. The hybrid beams with general motion in the plane, the beams with only one axis of symmetry, and the rotating disk-beam system are controlled in Refs. [15–17], respectively. Stabilization of beams in three-dimensional space is considered in Refs. [18–21]. The control of Euler-Bernoulli beam with arbitrary decay rate is studied in Ref. [22]. Stabilization of interconnected system of Euler – Bernoulli beam and heat equation is found in Refs. [23,24]. The control of a circular curved beam is addressed in Ref. [25].

Although not investigated as extensively as the control of Euler-Bernoulli beam, the vibration control of Timoshenko beams has been pursued by some researchers. The control laws in early works were mainly linear boundary feedbacks that were used to achieve strong (asymptotic) or uniform (exponential) stability [26,27]. The control of Timoshenko beam including tip mass was addressed in Refs. [7,28]. The Timoshenko beam under external disturbances was stabilized in Refs. [29,30]. Timoshenko beams with output constraint, input dead-zone, input backlash, and interior delay were investigated in Refs. [31–34], respectively. The control of Timoshenko robotic arm was considered in Refs. [35–38].

This paper considers vibration control of Timoshenko beams with external disturbances. The approach adopted is based on designing a high-gain observer to estimate disturbances. Indeed, observer-based design tools have proven effective in controlling wide groups of uncertain systems [39]. The works having the most relevancy to this paper are found in Refs. [29,30]. The control in Ref. [29] consists of a discontinuous control law that is obtained using a Lyapunov-redesign strategy. The well-posedness of the closed-loop system is studied using the semigroup theory, and the uniform stability is shown through

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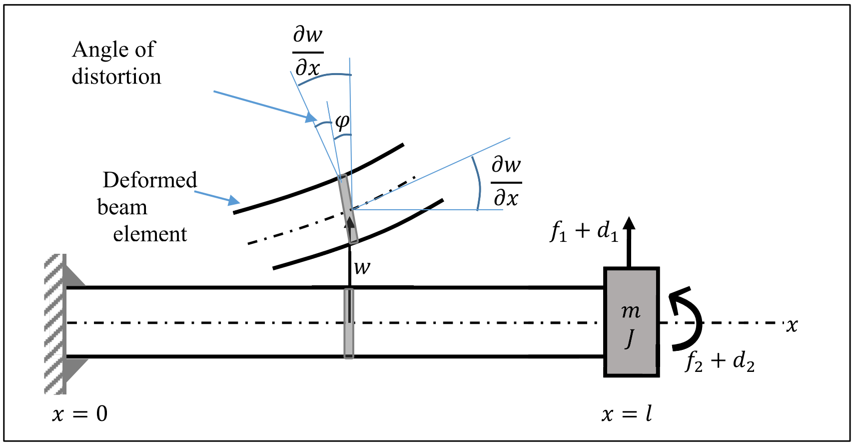

Fig. 1. Timoshenko beam: the cross section rotates by φ. which, unlike normal Euler–Bernoulli beam theory, is not equal to $\frac{\partial w}{\partial x}$.

Lyapunov analysis. On the other hand, the control in Ref. [30] is based on disturbance observer design. It is proved that the system response under the control proposed in Ref. [30] is bounded, but the well-posedness study is left out.

The focus of this paper is ADRC design for Timoshenko beam model including a tip mass with studying the well-posedness and stability of the model. The proposed control is a simple linear feedback law combined with a high-gain ESO that uses the tip mass dynamics to estimate the unknown disturbances. Considering the previous works, the main contributions of the paper are determined as follows.

- Compared with [29], which considers a discontinuous control law that is designed conventionally based on a worst-case strategy, this paper utilizes the ADRC to achieve a smooth control law that works based on the estimated values of disturbances. Therefore, as is typical of ADRC, the control proposed in this paper generates a more realistic control action and avoids the chattering. Moreover, it enables exact determination of disturbances and needs no prior knowledge of disturbance upper-bounds.
- Compared with [30], this paper proposes a new control law that is designed based on the approach established in Ref. [40] to guarantee the closed-loop model is well-posed. The well-posedness is proved using the semigroup theory. Next, to conclude that the system response is bounded under the external disturbances, it is shown using a frequency domain approach that the semigroup generated by the system operator is exponentially stable. The stability of the closed-loop system is further studied by the Lyapunov method.

The remainder of the paper is organized as follows. The next section describes the dynamical model. The proposed high-gain ESO is presented and analyzed in Section 3. Section 4 presents the ADRC. The well-posedness of the closed-loop system is proved in Section 5. The Lyapunov stability is analyzed in Section 6. Sections 7 and 8 are devoted to simulation results and concluding remarks, respectively.

2. Timoshenko beam model

There are two fundamental theories for dynamic analysis of beam structures, namely Euler-Bernoulli and Timoshenko beam models. The standard Euler-Bernoulli beam theory is successful at predicting the flexural behavior of structures whose thickness to length ratio is highly small. Such theory yields satisfactory results for situations where the wave lengths of the deformation are relatively large compared with the thickness of the beam [6]. When the beam becomes thicker or shorter, the advanced theory of Timoshenko yields more accurate model by taking into account shear deformation and rotational bending effects. Due to shear deformation, the beam undergoes an angle of distortion that makes the cross section rotate at an angle that is not equal to the centerline slope (See Fig. 1). Therefore, the total angle of beam centerline is sum of beam section rotation due to bending and angle of distortion due to shear effect. Since the model considered in this paper is the standard Timoshenko beam model, only the dynamic equations are given in this section and more detailed information about dynamic modelling can be found in Refs. [6,41,42].

The configuration of the controlled beam is shown in Fig. 1, where according to Timoshenko's theory the equation governing the beam vibration is as follows:

$$\begin{cases} \rho\ddot{w} - K(w'' - \varphi') = 0 \\ I_\rho\ddot{\varphi} - EI\varphi'' - K(w' - \varphi) = 0 \end{cases} \tag{1}$$

which is subjected to the boundary conditions

$$\begin{cases} w(0, t) = \varphi(0, t) = 0 \\ m\ddot{w}(l, t) + K[w'(l, t) - \varphi(l, t)] = f_1(t) + d_1(t) \\ J\ddot{\varphi}(l, t) + EI\varphi'(l, t) = f_2(t) + d_2(t) \end{cases} \tag{2}$$

where x and t are independent spatial and time variables, respectively, $w = w(x, t)$ and $\varphi = \varphi(x, t)$ are transverse deflection and bending slope of the beam, respectively, l is length, EI is bending rigidity, K is shear modulus, ρ is uniform mass per unit length, and I_ρ is uniform mass moment of inertia. For convenience, the notations $(\dot{.}) = \partial/\partial t$ and $(.)' = \partial/\partial x$ are used in this paper. The beam is clamped at one end ($x = 0$) while free to control at the other end ($x = l$), and carries the tip payload with mass m and inertia J at the controlled end. The transverse force f_1 and bending moment f_2 are the control inputs applied at the beam tip, and the external disturbances d_1 and d_2 add error to f_1 and f_2, respectively.

3. High-gain ESO design and stability analysis

The efficiency of any ADRC scheme is largely influenced by its capability to estimate the disturbance, and hence by its ESO plan. In this section, an efficient boundary ESO is introduced in order to estimate disturbance in Timoshenko beam model. The main idea is to treat the disturbance as the additional state of the system and then to synthesize a high-gain observer in order to estimate the augmented system states [43]. The role of high-gain observer is to compensate the unknown dynamics of external disturbances [44,45].

The ESO proposed in this paper is achieved by defining the auxiliary variables $\eta_1(t)$ and $\eta_2(t)$ as follows

$$\begin{cases} \eta_1(t) = \dot{w}(l, t) + \beta_1[w'(l, t) - \varphi(l, t)] \\ \eta_2(t) = \dot{\varphi}(l, t) + \beta_2\varphi'(l, t) \end{cases} \tag{3}$$