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Practice article

Robust fuzzy tracking control of a quad-rotor unmanned aerial vehicle based on sector linearization and interval matrix approaches

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ABSTRACT

In this paper, the robust H_∞ fuzzy tracking control strategy for a quad-rotor unmanned aerial vehicle (UAV) with strong coupling and highly nonlinear is put forward based on the Takagi-Sugeno(T-S) fuzzy error model. Firstly, the quad-rotor UAV system is divided into altitude subsystem, position subsystem and attitude subsystem. Through selecting appropriate premise variables, three T-S fuzzy error models, which are equivalent to the error dynamic model, are established by the sector linearization approach. Next, the uncertainties in drag coefficients, moments of inertia are taken into account, and the interval matrix is introduced to describe them in altitude, position and attitude T-S fuzzy error models. Then the robust H_∞ fuzzy feedback controllers are designed to stabilize T-S fuzzy subsystems. Besides, according to the Lyapunov stability theorem, it is obtained that the LMI sufficient conditions of exponential stability with the prescribed H_∞ performance for T-S fuzzy closed-loop subsystems. Meanwhile, the method for solving the gain matrices of controller is presented. Finally, simulation results are given to demonstrate the effectiveness, robustness and advantages of the proposed method. Then the feasibility of the proposed method is further verified by experimental results.

1. Introduction

As we all know, the quad-rotor unmanned aerial vehicle (UAV) is of small size, vertical take-off, autonomous hover and etc., and it has been widely used in military and civilian in recent years [1,2]. However, there is a big challenge to design controller for the quad-rotor UAV due to its strong couplings, highly nonlinear, parameter uncertainties, as well as external disturbances [3,4]. Therefore, the research on the control problem of the quad-rotor UAV has attracted wide attention of scholars at home and abroad.

In the past years, many studies have been done to solve the problem of strong couplings and highly nonlinear in the dynamic model of physical system [5,6] and a lot of significant results have been obtained on quad-rotor UAV system. In Refs. [4,7], the backstepping method is applied to realize trajectory tracking, but the tracking algorithm is complicated and difficult to implement. In Ref. [8], the nonlinear terms in the dynamic model of quad-rotor UAV are treated as uncertainties, but this treatment method decreases the model precision. In Refs. [9–11], the inner and outer loop approach is employed to deal with strong coupling and highly nonlinear in the dynamic model of quad-rotor UAV and achieve good flight performance. However, if the inner loop can not achieve fast and accurate tracking of the desired trajectory, the tracking performance of the outer loop will be seriously affected. It comes to light that the T-S fuzzy model based on sector linearization method is received wide application because it is an effective tool to describe the complex nonlinear systems with a set of local linear models [12–15], besides, it is very convenient to make use of the linear control theory to design control strategy for T-S fuzzy system. As far as we know, few literature, in which the T-S fuzzy model is adopted to study the trajectory tracking of the quad-rotor UAV, are reported. The fuzzy state models based on sector linearization approach for the attitude subsystem in Refs. [16,17] and the whole system in Refs. [18,19] are established. In addition, the controllers are designed to stabilize the quad-rotor UAV. However, the drag coefficients and the moments of inertia are ignored or the uncertainty in them are not taken into consideration, these controllers can not guarantee the tracking performance and they are only effective at low speed flight.

During the flight at medium or high speed, the aerodynamic's effect is significant and its effect of attitude control seriously relies on the moments of inertia [20–22]. Thus, the uncertainties in drag coefficients and the moments of inertia can cut down or affect the dynamic performance of quad-rotor UAV and different approaches have been applied to improve its robustness. In Refs. [9,23], an *I&I*-based adaptive method is used to estimate the uncertain drag coefficient. [24] uses the neural network to approximate to the uncertain aerodynamic drag coefficient. In Ref. [22], the adaptive law is used to estimate the

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moments of inertia. In Ref. [25], a nonlinear H_∞ controller is designed with an amount of ± 20% in the uncertain moment of inertia. But all the uncertain parameters in the dynamic model of the quad-rotor UAV are not fully discussed in Refs. [9,22–25]. As a matter of fact, the uncertainties in drag coefficients, lift coefficients and the moment of inertia exist simultaneous. In addition, drag coefficients and lift coefficients vary with the changes of air density, temperature, and etc., they all have their own variation range. The moment of inertias also have a certain range due to the existence of measurement errors. That is to say all these parameters have respective upper and lower bounds. Thus, we can introduce interval variables to describe these uncertain parameters. When the quad-rotor flies outdoors, the external disturbances always exist, thus, the H_∞ feedback controller should be designed to restrain them.

Based on the above investigation, the robust H_∞ fuzzy feedback controller based on the sector linearization and interval matrix approaches is put forward to track the desired trajectory for the quad-rotor UAV in this paper. The main contributions of this paper lie in: 1) through selecting appropriate premise variables, three T-S fuzzy error models are established to represent the altitude, position and attitude error subsystem, respectively. 2) interval variable is introduced to describe the parameter uncertainties in the T-S fuzzy error models. 3) the robust H_∞ fuzzy feedback controllers are designed by considering external disturbances, as well as the incomplete feedforward compensation, and the LMI based algorithm to obtain gain matrices are also provided. 4) simulation results are given to demonstrate the effectiveness, robustness of the proposed method and the advantages over the inner and outer loop robust H_∞ feedback approach. Besides, experimental results are presented to further show that our proposed robust H_∞ fuzzy controller is feasible.

This paper is organized as follows. First, the dynamic model of quad-rotor UAV is introduced in Section 2. In section 3, three T-S fuzzy error models are established based on the sector linearization method. The robust H_∞ fuzzy feedback controller, as well as the LMI-based algorithm to obtain gain matrices are proposed in section 4. In Section 5, simulation comparisons and experimental results are presented to illustrate control performance. Finally, conclusions are made in Section 6.

2. The dynamic model of quad-rotor UAV

The structure of quad-rotor is depicted in Fig. 1. F_1, F_2, F_3, F_4 are the lift forces generated by the four rotors, respectively. Let $\{I, X_I, Y_I, Z_I\}$ represent the right-handed inertia frame attached to the ground and Z_I toward the sky. The body fixed frame is denoted by $\{B, X_B, Y_B, Z_B\}$ with origin located at the mass center of the quad-rotor. $\{x(t), y(t), z(t)\}$ represents the position in the right-handed inertia frame, and its attitude is denoted by three Euler angles as $\{\phi(t), \theta(t), \psi(t)\}$.

Taking the external disturbances into consideration, the dynamic model of the quad-rotor UAV expressed in I is as follows [3,4].

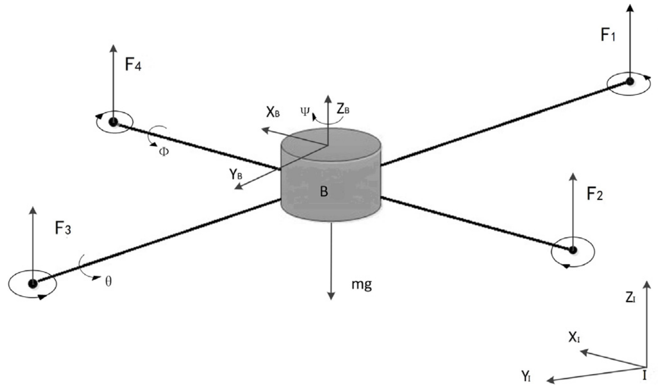

Fig. 1. The schematic diagram of quad-rotor UAV.

$$\ddot{x} = -\frac{K_x}{m}\dot{x} + (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\frac{u_t}{m} + d_x(t) \tag{1}$$

$$\ddot{y} = -\frac{K_y}{m}\dot{y} + (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\frac{u_t}{m} + d_y(t) \tag{2}$$

$$\ddot{z} = -\frac{K_z}{m}\dot{z} + (\cos\phi\cos\theta)\frac{u_t}{m} - g + d_z(t) \tag{3}$$

$$\ddot{\phi} = -\frac{K_\phi}{I_x}\dot{\phi} + \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\psi} - \frac{I_r}{I_x}\Omega\dot{\theta} + \frac{l\tau_1}{I_x} + d_\phi(t) \tag{4}$$

$$\ddot{\theta} = -\frac{K_\theta}{I_y}\dot{\theta} + \frac{I_z - I_x}{I_y}\dot{\phi}\dot{\psi} - \frac{I_r}{I_y}\Omega\dot{\phi} + \frac{l\tau_2}{I_y} + d_\theta(t) \tag{5}$$

$$\ddot{\psi} = -\frac{K_\psi}{I_z}\dot{\psi} + \frac{I_x - I_y}{I_z}\dot{\phi}\dot{\theta} + \frac{c\tau_3}{I_z} + d_\psi(t) \tag{6}$$

where, m is the mass of the quad-rotor; K_x, K_y, K_z, K_ϕ, K_θ, K_ψ are the corresponding drag coefficients; I_x, I_y, I_z are the moment of inertia each axis; I_r is the moment of inertia of rotor; $\Omega = w_4 + w_3 - w_2 - w_1$; w_i ($i = 1, \ldots, 4$) represents the rotational speeds of the four rotors on the quad-rotor UAV; g is the acceleration of gravity; c is the force and torque proportional coefficient; l is the distance from the center of each rotor to the center of mass; u_t, τ_1, τ_2 and τ_3 denote the total lift force, the roll force, the pitch force and the yaw force, respectively, and their relationship with the four rotor lift force is as follows.

$$\begin{aligned} u_t &= F_1 + F_2 + F_3 + F_4; \quad \tau_1 = -F_1 - F_2 + F_3 + F_4; \\ \tau_2 &= -F_1 + F_2 + F_3 - F_4; \quad \tau_3 = F_1 + F_2 - F_3 + F_4. \end{aligned}$$

where, $F_i = K_l w_i^2$, ($i = 1,\ldots,4$) and K_l is the positive and constant speed-to-force coefficient, which is related to the density of air, the radius of the propeller, and the geometry, lift and drag coefficients of the blade [26].

It is worth mentioning that although the drag coefficients K_x, K_y, K_z, K_ϕ, K_θ, K_ψ and c in the dynamic model vary with the changes of air density, temperature, and etc., they all have their own variation range. In addition, the moment of inertias I_x, I_y and I_z also have a certain range due to the existence of measurement errors [20,27,28]. That is to say all these parameters have respective upper and lower bounds.

In the sequel, a useful definition and lemma, which are needed to solve our problem, are introduced.

Definition 1. [29] *If $A \in [A^m, A^M] = \{[a_{ij}]: a_{ij}^m \le a_{ij} \le a_{ij}^M, 1 \le i, j \le n\}$, then A is called an interval matrix. Where $A^m = [a_{ij}^m]_{n\times n}$, $A^M = [a_{ij}^M]_{n\times n}$, for $1 \le i, j \le n$, satisfy $a_{ij}^m \le a_{ij}^M$.*

Lemma 1. [29] *For a given interval matrix $A \in [A^m, A^M]$, A can be described as*

$$A = A_0 + E_a\Sigma_a F_a \qquad \Sigma_a \in \Sigma_a^*$$

where, $A_0 = \frac{1}{2}(A^M + A^m)$;

$$\begin{aligned} E_a &= [\sqrt{\varsigma_{11}}e_1\cdots\sqrt{\varsigma_{1n}}e_1\cdots\sqrt{\varsigma_{n1}}e_n\cdots\sqrt{\varsigma_{nn}}e_n]_{n\times n^2}; \\ F_a &= [\sqrt{\varsigma_{11}}e_1\cdots\sqrt{\varsigma_{1n}}e_n\cdots\sqrt{\varsigma_{n1}}e_1\cdots\sqrt{\varsigma_{n}}e_n]^T_{n^2\times n}; \\ \Sigma_a^* &= \{\mathrm{diag}\{\delta_{11}\cdots\delta_{1n}\cdots\delta_{n1}\cdots\delta_{nn}\} \in R^{n^2\times n^2}, \\ |\delta_{ij}| &\le 1, i, j = 1,\ldots,n\}; \end{aligned}$$

$\varsigma_{ij} = (a_{ij}^M - a_{ij}^m)/2$, $(i, j = 1,\ldots,n)$; $e_i(i = 1, \ldots, n)$ *donates the ith column vector of the identity matrix. Obvariously, for* $\forall\, \Sigma_a \in \Sigma_a^*$, *there has* $\Sigma_a^T\Sigma_a \le 1$.

Remark 1. *All the states$\{x, y, z, \phi, \theta, \psi\}$ and u_t in the quad-rotor UAV system are bounded, and the selected premise variables are also bounded because all of them are the function of state variables or u_t, which are bounded. In addition, the upper and low bounds of the premise variables don't include the singular values of the quad-rotor UAV.*