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Research article

A novel approach of fractional-order time delay system modeling based on Haar wavelet

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ABSTRACT

In this paper, fractional-order time delay system modeling is presented using Haar wavelet operational matrix of integration. Therefore, it does not require any prior knowledge of transfer function structure or partial information about fractional differentiation order. It allows the estimation of the implicit time delay parameter together with other model parameters by utilizing new delay operational matrix of Haar wavelet based on Riemann-Liouville definition. The proposed technique reduces the complexity of identification by converting the complex fractional calculus equations into simple algebra. The efficacy of the approach is verified on various integer and non-integer (fractional) order systems in simulation. For realistic condition, proposed method is verified in the presence of noise in simulation and also demonstrated on the real-time process control temperature system.

1. Introduction

Fractional calculus (FC) is a mathematical analysis concept that is more than three centuries old, having been discussed first in a letter between the mathematicians Leibniz and L'Hospital in 1695 [1]. Initially confined to pure mathematics, in recent years, FC has been increasingly applied in many engineering domains [2–4]. Many physical systems, like semi-infinite lossy transmission line, dielectric polarization, viscoelasticity, diffusion of heat into semi-infinite solid, colored noise, electrode-electrolyte polarization, boundary layer effects in ducts and electromagnetic waves, are better understood with fractional order models (FOMs) [3,5–7].

Fractional order modeling has been used for various applications. Modeling of Supercapacitor [8–10], fractional filter design [11], fractional oscillators [12], biological model [13], thermo-mechanical behavior [14], atmospheric dispersion [15], lithium-ion batteries [16], acid-lead battery [17] have been well explained using FOMs.

Various techniques for system approximation have been developed in both time and frequency domain with FOMs. Poinot and Trigeassou [18] employed output error technique to estimate parameters and fractional order (FO) of real fractional heat transfer system using state-space representation. Djamah et al. [19] developed low order identification method for fractional dynamic systems using the fractional integration operator. The fractional system was approximated by an integer state-space representation of high dimension. Li and Sun [20] developed a method for the solution of FO differential equation using generalized block pulse operational matrix. Parameter and differentiation order estimation method was described in Ref. [21] using simplified refined instrumental variable method and a gradient based algorithm. Tang et al. [22] applied generalized block pulse operational matrix for parameter identification of FO systems. Early works on application of Haar wavelet operational matrix are given in Refs. [23–25]. Li et al. [24] demonstrated a novel method using the operational matrix to compute the fractional order differentiation (FOD) of a signal through expanding the signal by the Haar wavelets and constructing Haar wavelet operational matrix of the FOD. Later, they explored the same concept of Haar wavelet operational matrix for the parameter identification of FO linear systems [25]. In order to improve the efficiency of the parameter identification, a frequency domain method was proposed in Ref. [26] using variable damping least square approach. Fractional first order system identification method for super-capacitor charging circuit using discrete-time Laguerre-based approximation was described in Ref. [8]. Dai et al. [27] described modulating function based method for identification of FO system using recursive least squares estimation.

All the aforementioned work involved FO systems without time delay, which is commonly observed in chemical, electronic, mechanical and biological systems. It is challenging work to approximate and

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control such systems. In recent years, several novel techniques have been proposed. Using harmony search and linear least squares approach, a frequency domain identification method for FO time delay systems was proposed in Ref. [28]. In Ref. [29], the Nyquist based model reduction technique is used for a reduced parameter modeling of the FOM with time delay. Narang et al. [30] proposed an identification method using linear filter for FOM with time delay. Ahmed [31] presented an integral equation approach (IEA) for FOS modeling using step response data assuming that the fractional orders were known. In this method prior knowledge of nearest integer of the highest order in the denominator is required. Also for the models without zero, some knowledge of delay is required for identification. Nie et al. [32] proposed techniques using three points data (TPD) on the step response and single-variable search (SVS). More recently, block pulse functions (BPF) were used by Tang et al. [33] to make block pulse operational matrices (BPOM) for FOS with time delay. In this method, all unknown model parameters can be identified simultaneously through minimizing a quadric error function.

Literature shows certain challenging issues in FOS identification. Firstly, simultaneous estimation of parameters and time delay is relatively complex in calculation and for that some methods have used partial information about FOD. In order to complete exact model identification, algorithm usually requires previous knowledge or structure of transfer function. Another issue is associated with real time verification and with noisy data. It is also required to see how these methods deliver result in case to model higher-order dynamics into the low-order.

The operational matrices, which can be obtained using any orthogonal basis function such as a block pulse and Haar wavelet, can convert the complex integral-differential equations of any fractional orders into simple algebraic equations. It has given overall more simplicity and accurate estimation without much computational complexity. In most reported methods, including those involving orthogonal basis functions, the time delay estimation is treated separately from the estimation of model parameters.

Most studied so far for FOS are either only those with no time delays or those that require some initial information to estimate the parameters of the system model. So, how to identify the FOS with time delay is still an open problem. This is an attempt, via Haar wavelet, to estimate both model and time delay parameters together from output signal data generated after a single step or random input excitation. It is shown that the properties of Haar wavelet can obtain the system model more accurately especially without fractional derivatives of input and output signals.

In this work, FOS system is considered without having previous knowledge of its integer order (IO) model or structure of a transfer function. Therefore, different input excitation is used to generate output response data. Using input-output data FO model for considered system is obtained. A method is completely established on Haar wavelet operational matrix and Riemann-Liouville (R-L) fractional integrals and derivatives formulae. Application of operational matrices reduces complexity and consequently, facilitates simultaneous estimation of delay, differentiation orders and other model parameters. The method converts the complex FC calculation into simple algebraic equations, so it reduces further computational time and convergence problem. Numerical study and experimental result verify the validity of the proposed method. The remainder of the paper is organized as follows. The mathematical background of Haar wavelet and operational matrices are firstly introduced in Section 2. The following Section 3 details the identification methodology for FOS followed by numerical simulation study and practical verification in Section 4 and finally conclusion is drawn in Section 5.

2. Mathematical background

2.1. Fractional calculus

Fractional calculus is a generalization of non-integer (real) order integration and differentiation and its operator is generally defined as

$$ {}_aD_t^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & \alpha > 0; \\ 1 & \alpha = 0; \\ \int_a^t (d\tau)^{-\alpha} & \alpha < 0; \end{cases} \tag{1} $$

where a and t are the bounds of the operation and α ($\alpha \in R$) is the order of operation. There exist numerous definitions to characterize fractional integration and differentiation. In our work, we use the R-L definition which can be written as

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \tag{2} $$

where $n-1 < \alpha < n$, $n \in N$, and Γ denotes gamma function. Suppose the FO derivative is represented in the Laplace domain with zero initial. Then from Ref. [4],

$$ L[{}_0D_t^{\alpha} f(t)] = s^{\alpha} F(s) \tag{3} $$

where s^{α} is a fractional Laplacian operator.

The fractional R-L integration of an arbitrary function $f(t)$ is given by

$$ (I_a^{\alpha} f)(t) = \frac{1}{\Gamma(\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \tag{4} $$

By using convolution property, (4) can be simplified further to

$$ (I_a^{\alpha} f)(t) = \frac{1}{\Gamma(\alpha)} t^{\alpha-1} * f(t) \tag{5} $$

where $t > a$ and α is the real positive integration order and $*$ denotes convolution. It can also be written in the Laplace domain with zero initial condition as

$$ L[I_0^{\alpha} f(t)] = \frac{1}{s^{\alpha}} F(s) \tag{6} $$

2.2. Haar wavelet

Haar wavelets are piecewise constant functions and utilized in this work due to their high accuracy, mathematical simplicity, and their ease of implementation with other standard algorithms. Computationally, they are faster compared to other functions of wavelet family. Haar wavelets have the advantage of noise immunity. One can set the threshold for Haar wavelet coefficients and minimize the effect of noise [24]. This is one of the most important features for the system identification. Haar wavelets are defined as,

$$ h_i(t) = \frac{1}{\sqrt{m}} \begin{cases} 2^{\frac{\delta}{2}} & \frac{r}{2^{\delta}} \leqslant t < \frac{r+0.5}{2^{\delta}} \\ -2^{\frac{\delta}{2}} & \frac{r+0.5}{2^{\delta}} \leqslant t < \frac{r+1}{2^{\delta}} \\ 0 & \text{elsewhere} \end{cases} \tag{7} $$

where $i = 1, 2, .., (m-1)$ gives the series index number and $m = 2^{\nu}$ defines the resolution. An δ and r represent the integer decomposition of the index i. Thus, $i = 2^{\delta} + r$ where $\delta = 0, 1, r, .., (\nu-1)$, $r = 0, 1, 2, \ldots, (2^{\delta}-1)$ and $m, i, r \in Z$. It is noted that $h_0(t) = \frac{1}{\sqrt{m}}$ for $0 \leqslant t < 1$, and 0 otherwise. Also,

$$ h_1(t) = \frac{1}{\sqrt{m}} \begin{cases} 1 & 0 \leqslant t < \frac{1}{2}; \\ -1 & \frac{1}{2} \leqslant t < 1; \\ 0 & \text{otherwise.} \end{cases} \tag{8} $$

An arbitrary function $x(t) \in L^2([0, 1))$ can be written for first M terms only in Haar wavelets as