ISA Transactions xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Precise position control of an electro-hydraulic servo system via robust linear approximation

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ARTICLE INFO

Keywords:
Electro hydraulic
Linearization
Overall Convex Mapping(OCM)
Polytopic uncertainty

ABSTRACT

This paper presents a study on electro-hydraulic servo system for the purpose of position control using a compatible linear model. The system has high level of nonlinearity and linearization introduces extra error in system model. In order to reduce this error several methods of linearization uncertainty are discussed. In spite of applying Taylor's series for all methods, several procedures are used for considering uncertainty on linearization constants. In the first procedure, a simple bound is considered for each linearization constant. In the second procedure, a polytope is extracted for the uncertainty by a graphical method. Finally, a procedure with less conservativeness and less restriction is proposed. This procedure is used to extract the linear model of the electro-hydraulic servo system for the task of position control. The resulting model is used to synthesize an output-feedback H_∞ controller for the EHSS using a Linear Matrix Inequality (LMI)-based approach. The effectiveness of the proposed method is demonstrated by simulation and experimental results. The results showed that the procedure is less conservative and has the fastest operation without any overshoot.

1. Introduction

Fluid power systems have widespread industrial applications. Important properties such as high power to weight ratio and suitable reliability and controllability have made fluid power systems applicable in cases such as industrial heavy-duty machines [1]. Electro-hydraulic servo system(EHSS) is excellent for position control purposes. But the equation of the system has extreme level of non-linearity. This nonlinearity combines with unmodeled dynamic such as friction and leakage and increase the complexity of system control.

Complex nonlinear methods have been used to overcome the mentioned nonlinearity and mis-matched model in hydraulic and pneumatic position control systems by Refs. [2–5]. Internal leakage between the chamber of the cylinder causes positioning error and Refs. [6,7] have focused on detection and eliminating the resulted error by extended nonlinear mapping and controlling method.

The objective of this paper is linearizing electro-hydraulic servo system with a mission of position control in a novel manner that can describe its behavior exactly.

A wide number of control systems are modeled with nonlinear equations. A common practice is linearizing around an operating point and design a stable control law for the linear system. Albeit, there are some essentially nonlinear phenomena such as chaos and limit cycle that can take place only in the presence of nonlinearity [8].

Taylor's series as the most common tool, linearizes the equation around an operating point of systems which involves some error to the problem. When the actual system variables are far from the operating point, the linearized system will fail to model the behavior of the system. Some researchers have concentrated on reducing the error as much as possible.

Several works have been carried out in the past such as Pseudo-linearization, Frozen input method, Velocity-based method and generalized input-output injection method were concluded and extended afterward. A necessary and sufficient condition for the MIMO system linearization by generalized state condition transformations and generalized input-output injection has been obtained by Plestan and Glumineau [9]. This procedure eliminates the state from system equations and is suitable only for nonlinear observers depending on input or output derivatives.

Some linearization methods have been suggested for time varying systems. Frozen input theory is derived by representing a small neighborhood of equilibrium point. This method has inherent restrictions

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https://doi.org/10.1016/j.isatra.2018.06.002
Received 24 June 2017; Received in revised form 16 May 2018; Accepted 1 June 2018

such as small vicinity about the equilibrium operating point. Leith and Leithead have proposed a family of velocity-based linearization for a nonlinear system [10]. The procedure can be applied to the analysis gain-scheduled systems as is done in Ref. [11]. In spite of facilitating dynamic analysis far from the equilibrium point, a slow variation condition is required in a velocity-based method. Toivonen et al. has applied internal model control on nonlinear systems which are modeled as linear parameter-varying systems based on velocity-based linearization [12].

Althoff et al. have represented the linearization error as a bounded uncertain parameter evaluated by Lagrange reminders [13]. A reachable set of states for a nonlinear system with uncertain parameters and inputs is computed in the paper.

Although many linearization methods have been suggested by mathematical researchers, they have not put into practice. This may be raised from the difficulty of applying, unfamiliar basis or incompatibility with control design procedures. Milic et al. considered linearization error as a parametric and unstructured uncertainty for a position robust controller in an EHSS setup [14]. Despite of no discussion about how the limit of this parameter is extracted, there is a good idea of considering the error in linearization constant. Due to parametric representation of uncertainty, large conservativeness is considered in the problem. This procedure is not limited to linearization error and can be used to determine the range of each nonlinear function existing in problem. The nonlinear term can also be considered as an uncertain constants, as [15] reported.

Polytope uncertainty is among the most widely used procedures of uncertainty representation in many robust linear problems of control design [16–18]. This tool is used for representing linearization uncertainty in a force control and position control task of an EHSS setup respectively by Refs. [19] and [20]. In the works, each linearization constant is figured for a set of system parameter and a convex polytope is encircled around it separately. In order to determine linearization constants, a combination of polytope vertices are considered as uncertainty. In spite of reducing the conservativeness in comparison of [14], there are some restrictions in this procedure.

Conservative design reduces the possibility of high speed gain and increases the difficulty of optimization problems. Therefore, efforts have been made to reduce conservativeness in the modeling and design process of control systems. In Ref. [21], a state-feedback controller and parameter-dependent Lyapunov function provides a less conservative controller for cases such as mechanical vibration systems. Enhanced H_2 LMI constraints for a eigenstructure assignment is reported by Ref. [22]. LMI optimization conditions for stability of discrete-time systems with slope-restricted nonlinearities has proposed in Ref. [23]. For asymptotic stability of impulsive systems [24], represented a new less conservative conditions.

The aim of this work is to extend the procedure represented by Refs. [19] and [20] in order to reach a general and less conservative procedure. In this approach, all linearization constants are considered as an element of the vector function and the conservativeness obtained by combination is disappeared. Moreover, there is no requirement of figuring, and the restriction of the variable number is deleted. As a result the linearization error can be modeled in a neater way. This procedure can also be used to find an uncertain parameter instead of nonlinear term. The procedure is presented in a manner that can be applicable for any system, and then applied on EHSS with the task of position control. All the mentioned procedures for considering linearization error are applied and uncertain linear models are extracted. To address uncertain model control, an H_∞ robust control is synthesized by the Linearization Matrix Inequality (LMI) method for each situation and experimental results are reported.

2. Preliminaries and problem formulation

The following nonlinear time-invariant system is considered:

$$\dot{x} = f(x, u) \tag{1}$$

Where $x \in R^n$ is the state vector, $u \in R^m$ is the control signal and f is an arbitrary function including nonlinear terms. For linearization, f: $D \to R^n$ must be as a continuously differentiable function and D as a neighborhood of the operating point. By expanding f using the Taylor's series about (x^*, u^*) and ignoring high order terms, the following linear system will be obtained:

$$\begin{aligned}\dot{x} \;&\approx f(x^*, u^*) + \frac{\partial f(x,u)}{\partial x}\Big|_{\substack{x=x^*\\u=u^*}}(x - x^*)\\ &\quad + \frac{\partial f(x,u)}{\partial u}\Big|_{\substack{x=x^*\\u=u^*}}(u - u^*)\end{aligned} \tag{2}$$

Considering (x^*, u^*) as an equilibrium points concludes to $f(x^*, u^*) = 0$ and by substituting $(u - u^*)$ with u and $(x - x^*)$ with x, (2) can be expressed as a standard linear system through the following equation.

$$\dot{x} = Ax + Bu; \quad A \in R^{n\times n}, \quad B \in R^{n\times m} \tag{3}$$

Where A, B matrices are defined as the following terms

$$A = [a_{ij}] = \left[\frac{\partial f_i(x,u)}{\partial x_j}\Bigg|_{\substack{x=x^*\\u=u^*}}\right]; \; i, j = 1, \ldots, n \tag{4}$$

$$B = [b_{ik}] = \left[\frac{\partial f_i(x,u)}{\partial u_k}\Bigg|_{\substack{x=x^*\\u=u^*}}\right]; \; k = 1, \ldots, m \tag{5}$$

In fact, the nonlinear function is approximated by a tangent plane crossing the operating point. The more the variables of the actual system deviate from the operating point, the more prominent the linearization error is.

3. Linearization uncertainty

A method to reduce linearization error is to put uncertainty in linearization constants. There are several methods for defining the uncertainty, which will be discussed in the following sections. However before that, there must be some definition to simplify methods representation. Linearization constants are A, B elements extracted by f elements deviation as (4), (5) showed. Each element of the f can be either a nonlinear or a linear function of x and u or be a constant parameter. J_r, $r = 1, \ldots, \alpha$ is defined as the symbol of each non-constant term which is extracted by f elements deviation and will be utilized as intermediate function.

$$\begin{aligned}J_r \quad &= \left\{\frac{\partial f_i(x,u)}{\partial x_j} \;\; or \;\; \frac{\partial f_i(x,u)}{\partial u_k}\,\middle|\, \frac{\partial f_i(x,u)}{\partial x_j} \neq cte \;\; or\right.\\ &\left.\frac{\partial f_i(x,u)}{\partial u_k} \neq cte \;\;, i = 1, \ldots, n \;\;, j = 1, \ldots, n \;\;, k = 1, \ldots, m\right\}\end{aligned} \tag{6}$$

p_s, $s = 1, \ldots, \beta$ is also considered as the symbol of variables, which are some elements of x and u. If the derivative of a f_i relative to x_i or u_k is constant, that is, f_i is linear relative to the variable. Therefore, according to the Taylor expansion, it is simply linearized and there is no need to replace it with a constant value with uncertainty. As a simple example, the following system can be considered:

$$f(x, u) = \dot{x} = -\frac{1}{2}\exp(-x^2 - u^2) + \cos(1 - u) \tag{7}$$

For this system, intermediate functions and their variables are defined as

$$\begin{aligned}J_1 \;&= \; \frac{\partial f}{\partial x} = x\exp(-x^2 - u^2)\\ J_2 \;&= \; \frac{\partial f}{\partial u} = u\exp(-x^2 - u^2) + \sin(1 - u)\\ p \;&= \; (p_1, p_2) = (x, u)\end{aligned}$$