Chinese Journal of Aeronautics, (2018), **xxx**(xx): xxx–xxx

Chinese Society of Aeronautics and Astronautics
& Beihang University

**Chinese Journal of Aeronautics**

cja@buaa.edu.cn
www.sciencedirect.com

# A thermal-flutter criterion for an open thin-walled circular cantilever beam subject to solar heating

**Xiaode YUAN, Zhihai XIANG** *

*Department of Engineering Mechanics, Tsinghua University of Aerospace Engineering, Beijing 100084, China*

Received 1 March 2018; revised 16 April 2018; accepted 15 May 2018

**KEYWORDS**

Fourier finite element;
Stability criteria;
Thermal flutter;
Thermally Induced Vibration (TIV);
Thin-walled structures

**Abstract** The flexible attachments of spacecraft may undergo Thermally Induced Vibration (TIV) on orbit due to the suddenly changed solar heating. The unstable TIV, called thermal-flutter, can cause serious damage to the spacecraft. In this paper, the coupled bending-torsion thermal vibration equations for an open thin-walled circular cantilever beam are established. By analyzing the stability of these equations based on the first Lyapunov method, the thermal-flutter criterion can be obtained. The criterion is very different form that of closed thin-walled beams because the torsion has great impact on the stability of the TIV for open thin-walled beams. Several numerical simulations are conducted to demonstrate that the theoretical predictions agree very well with the finite element results, which mean that the criterion are reliable.



## 1. Introduction

The flexible attachments of spacecraft generally have the characteristics of large size, light weight, low stiffness and small heat capacity. Therefore, these structures are prone to experiencing the Thermally Induced Vibration (TIV) due to the suddenly applied solar heat flux when the spacecraft enter or leave eclipse.[1–4] These vibrations could reduce the pointing accuracy of spacecraft and even introduce damage into the structure, especially when the vibration is unstable, i.e., the thermal-flutter.

TIV was firstly predicted theoretically by Boley as early as 1956.[5] Boley and Barber[6] showed that when a very thin beam or plate is subjected to rapid surface heating, the vibration can be induced by a kind of time-dependent thermal moment due to the rapid temperature gradient in the structure. Later on, the Boley parameter $B = \tau_T \omega_1$ was defined to characterize the severity of TIV for cantilever beams,[7] where $\tau_T$ is the thermal characteristic time and $\omega_1$ is the minimum angular frequency of the beam. The ratio of the maximum dynamic deflection over the quasi-static deflection of a cantilever beam can be expressed as $1 + 1/\sqrt{1 + B^2}$, which means that the smaller $B$ is, the more severe the TIV is. Although the Boley parameter $B$ is a nice index for pure bending TIV of a cantilever beam, practical structures may have more complex TIV modes. For example, the structure composed of open

* Corresponding author.
E-mail address: xiangzhihai@tsinghua.edu.cn (Z. XIANG).

Peer review under responsibility of Editorial Committee of CJA.

ELSEVIER **Production and hosting by Elsevier**

https://doi.org/10.1016/j.cja.2018.07.002
1000-9361 © 2018 Production and hosting by Elsevier Ltd. on behalf of Chinese Society of Aeronautics and Astronautics.


Please cite this article in press as: YUAN X, XIANG Z A thermal-flutter criterion for an open thin-walled circular cantilever beam subject to solar heating, *Chin J Aeronaut* (2018), https://doi.org/10.1016/j.cja.2018.07.002

thin-walled beams is apt to undergo torsional vibration due to its ultra-low torsional stiffness.[8]

Compared to stable TIV, the thermal flutter is more harmful to space structures. This phenomenon was first observed on orbit in 1968[9] and then it was realized in a laboratory environment.[10] After that, more and more coupled thermal-structure analyses were conducted to investigate the condition of thermal flutter.[11,12] Yu first established the stability criterion on the TIV of a closed thin-walled cantilever beam subject to solar heating[13] and then that criterion was updated by Graham.[14] In Graham's criterion, the thermal flutter will only happen.

when the beam axis points away from the sun, where the beam axis is defined as the vector pointing from the fixed end of the beam to the free end of the beam. An important conclusion of this criterion is that the normal-incident heat flux will not induce thermal flutter, which is contradictory to both experiment results[15] and numerical simulations.[16] Realizing that the stability analysis should be established on the deformed steady state instead of the original configuration of the beam, Zhang and Xiang proposed a new criterion, which conforms with the experimental and numerical results.[17]

All existing criteria of thermal flutter are only applicable to closed thin-walled beams. In contrast, the criterion for open thin-walled beams must consider the bending and torsion coupling deformations. Consequently, the circumferential incident angle of the heat flux should have great impact on the stability of the TIV. With a full consideration of these two points, this paper establishes a thermal-flutter criterion suitable for open thin-walled circular cantilever beams based on the first Lyapunov method.[18]

## 2. Coupled thermal-structural dynamic analysis

### 2.1. Analysis model and basic assumptions

As Fig. 1 shows, two sets of coordinate systems are defined to describe the deformation of the cantilever beam. $OXYZ$ is a fixed spatial coordinate system, in which $X$ axis is the initial centroid axis and $Y$ axis points to the initial opening direction. $Oxyz$ is a local coordinate system attached at a point on the beam, in which $x$ axis is the deformed centroid axis and $y$ axis always points to the opening direction of the rotated beam.

The dimensions of the interested beam are defined as follows: $l$ is the beam length; $R$ and $h$ are the midline radius and thickness of the beam cross-section, respectively. For a thin-walled slender beam, $h/R << 1$ and $R/l << 1$, so that Euler-Bernoulli beam theory is applicable.

The solar heat flux vector $\boldsymbol{S}_0$ is uniformly distributed along the beam length. $\theta_0$ is the angle between $\boldsymbol{S}_0$ and vector $\boldsymbol{n}$, which is the normal of the beam and opposite to the projection of $\boldsymbol{S}_0$ in plane $OYZ$. $\alpha$ is the angle between $\boldsymbol{S}_0$ and the $Y$ axis in the $YOZ$ plane.

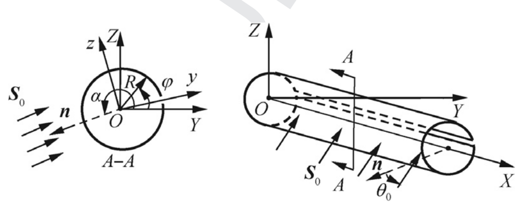


**Fig. 1** An open thin-walled circular cantilever beam subject to solar heat flux.

The following assumptions are adopted in the analysis:

(1) Emission and radiation of heat to space is considered but convection and radiation between the different surfaces of the beam are neglected.
(2) Heat transfer along beam length is neglected.
(3) At positions of $X = 0$, $X = l$ and the longitudinal opening sides of the beam are adiabatic.
(4) The amplitude of the perturbation temperature is much smaller than the average temperature in the cross-section.
(5) Damping is not considered.
(6) Deflections and rotations are small before fluttering.

### 2.2. Basic equations

Bending and torsion of an open thin-walled beam are initiated mainly by the temperature gradients due to the absorbed heat flux. At the same time, the deformation also affects the incident angle of the heat flux. When the beam deforms, the absorbed solar heat flux is calculated as

$$q(x,\varphi,t) = \alpha_s S_0 \delta \cos(\varphi - \alpha')\sqrt{1-\psi^2} \tag{1}$$

where $\varphi$ is the circumferential angle along the midline of beam cross-section; $\alpha_s$ is the absorptivity of beam surface; $S_0$ is the magnitude of solar heat flux $\boldsymbol{S}_0$; $\alpha' \in (0, 2\pi)$ denotes the equivalent circumferential incident angle; $\psi$ is the angle between $\mathbf{S}_0$ and the deformed axis of the beam, and

$$\begin{cases} \alpha' = \alpha - \theta_x \\ \psi = \cos\theta_0\cos\alpha\sin\theta_{zi}\cos\theta_{yi} - \cos\theta_0\sin\alpha\sin\theta_{yi} + \sin\theta_0\cos\theta_{zi}\cos\theta_{yi} \end{cases} \tag{2}$$

$\theta_x$ is the torsion angle; $\theta_{yi}$ and $\theta_{zi}$ are the bending angles of the centroid around $y$ and $z$ axis, respectively; $\delta$ is defined as

$$\delta = \begin{cases} 1 & 2n\pi - \frac{\pi}{2} \leqslant \varphi - \alpha' \leqslant 2n\pi + \frac{\pi}{2} \\ 0 & 2n\pi + \frac{\pi}{2} \leqslant \varphi - \alpha' \leqslant 2n\pi + \frac{3\pi}{2} \end{cases} \quad n = 1, 2, \ldots \tag{3}$$

Based on Assumption (2), the beam temperature $T(x,\varphi,t)$ is determined by

$$c\rho\frac{\partial T}{\partial t} - \frac{k}{R^2}\cdot\frac{\partial^2 T}{\partial\varphi^2} + \frac{\varepsilon\sigma}{h}T^4 = \frac{q(x,\varphi,t)}{h} \tag{4}$$

where $c$ is specific heat; $\rho$ is mass density; $k$ is thermal conductivity; $\varepsilon$ is the emissivity of beam surface; $\sigma$ is the Stefan–Boltzmann constant.

Eq. (4) is a strong nonlinear equation, which is difficult to solve. However, it can be decomposed into two very simple equations by using the Fourier finite element method,[16] which approximates the temperature $T(x,\varphi,t)$ as the sum of an average temperature $T_a(x,t)$ and three perturbation temperatures:

$$T(x,\varphi,t) \approx T_a(x,t) + T_{p1}(x,t)\cos\frac{\varphi}{2} + T_{p2}(x,t)\cos\varphi + T_{p3}(x,t)\cos\frac{3\varphi}{2} \tag{5}$$

Substituting Eq. (5) into Eq. (4) and integrating it over the cross-section with respect to $\varphi$, one can obtain two decoupled equations:



Please cite this article in press as: YUAN X, XIANG Z A thermal-flutter criterion for an open thin-walled circular cantilever beam subject to solar heating, *Chin J Aeronaut* (2018), https://doi.org/10.1016/j.cja.2018.07.002