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Research paper

A new algorithm of links labelling for the isomorphism detection of various kinematic chains using binary code

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ABSTRACT

Isomorphism of kinematic chains (KCs) has always been a critical issue for the researchers dealing with structural synthesis. Consequently, many researchers of repute presented various methods during the last eight decades for the KCs with either simple and/or multiple joints. Binary code is one of such various methods, but the major problem lies in the algorithm of links labelling, which becomes cumbersome, in particular, for large KCs. The paper presents a simple algorithm of links labelling used to find out a binary sequence which, in turn, provides a maximum binary code (chain invariant). The algorithm is tested for six, seven, eight, nine, ten, eleven, twelve and fifteen links with simple joints, seven and eight links KCs with multiple joints and finally, the Epicyclic gear trains (EGTs) with four, five and six links for its efficiency and reliability. The results are in full agreement with the references taken for the purpose. The paper discusses, in a unique way, the decoding of the binary codes of different KCs also.

1. Introduction

Isomorphism is one of the most elusive structural properties required to be identified at the conceptual design stage. It causes to duplicate structures and therefore, a lot of time is wasted in the design process. Many researchers of repute have continuously been working for the last eight decades to solve the problem and consequently, various methods came into existence. As mentioned earlier, binary code is one of such methods, but the major problem here is the algorithm of links labelling so as to find out either a maximum or a minimum binary code. Therefore, there has always been a need to develop an easy, effective and efficient algorithm and the presented method is a sincere effort.

Levai [1] discussed all the thirty four different types of simple planetary gear trains (PGTs) having two central gears, one or more planet gears and one arm. The author proposed two alphabets *P* (called positive diameter used to identify external planetary or central gears) and *N* (called negative diameter used to identify internal planetary or central gears) in a particular order to identify a planetary gear chain, and also discussed mathematical and graphical methods for analysing torque and force acting on different links. Buchsbaum and Freudenstein [2] discussed about the functional, schematic and graph representation of different *KCs* including Epicyclic gear trains (EGTs). Freudenstein [3] discussed the concept of Boolean algebra for the structural synthesis of EGTs. Manolescu [4] proposed an idea of graphitization along with joint simplification and dyad amplification for the transformation of Baranov trusses for the generation of planar KCs with simple joints.

Uicker and Raicu [5] compared the advantage of link-link form of the incidence matrix over link-joint form of the incidence matrix and represented a KC by a set of characteristic coefficients of the characteristic polynomial of the link-link

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square symmetric matrix of the chain. The authors used these identification codes to identify the isomorphism. Gibson and Kramer [6] proved that only twenty two basic spur planetary gear trains (PGTs) are possible instead of thirty four as given by Levai [1] and proposed a scheme of symbolic notations to identify the type of gear pair and ultimately the whole EGTs.

Yan and Hwang [7] proposed linkage path code for the detection of isomorphism of planar KCs with simple joints and turning pairs. Mruthyunjaya [8–10] developed a fully computerized methodology for the structural synthesis of KCs, applied to several cases for which solutions were either fully or partially available. This was to check the reliability of the computer program and to derive new and complete collection of ninety seven 10 links and 3- degree of freedom (dof) KCs with simple joints.

Ravisankar and Mruthyunjaya [11] presented, for the first time, a fully computerized method for the structural synthesis of gear KCs. Dube and Rao [12] proposed characteristic polynomial method using graph theory and distance matrix and they compared the characteristic polynomials of the matrices to detect isomorphism of KCs. Ambekar and Agrawal [13–15] proposed max code and min code for KCs with simple joints. They utilized upper triangular portion of the adjacency matrices of KCs to generate binary sequences and binary codes. The advantage of this method is its decodability, but the disadvantage is the computational complications when applied to complex KCs. Despite the examples of co-spectral graphs in graph theoretic literature it was believed that the characteristic polynomials are most likely to be unique for closed KCs without singular links and/or without any over constrained sub-chains, Examples of 10 links and 1-dof non-isomorphic chains by Mruthyunjaya and Balasubramanian [16] having co-spectral graphs, however, belies the hopes.

Tsai [17] proposed linkage characteristic polynomial for the topological synthesis of 1-dof EGTs with up to six links and the isomorphic graphs were identified by comparing the values of their corresponding linkage characteristic polynomials. Agrawal and Rao [18] proposed link-link variable characteristic matrix for representing a KC. The determinant of this matrix, called variable characteristic polynomial, is claimed to provide a powerful set of identification numbers for single as well as multi-loop KCs with revolute pairs.

Chieng and Hoeltzel [19] proposed a combinatorial approach for automatic mechanism sketching of planar KCs and EGTs. Kim and Kwak [20] proposed a graph enumeration theory for selecting non-isomorphic graphs. The Hamming number approach [21–22] is introduced for isomorphism identification, but the counter examples also are found against them. Cheng [23] proposed an idea of the contracted graph to generate simple KCs with simple and multiple joints. The authors worked out to find out non isomorphic basic KCs by removing degenerate and isomorphic chains. Hsu and Lam [24] presented the concept of canonical displacement graphs and rotation graphs to represent the kinematic structure of PGTs and identified the displacement isomorphism of planetary gear trains. These concepts avoided the formation of pseudo-isomorphism.

Shin and Krishnamurthy [25] proposed a standard code for the unique representation of pin-jointed KCs based on graph theory. Chu and Cao [26] proposed link's adjacent-chain table (ACT) method to identify isomorphism and number of inversions of KCs. The authors opine that the proposed method is much simpler than the traditional methods of matrix. Yadav et al. [27–28] presented a sequential three-step test for isomorphism and proposed a link-link distance matrix approach. Yadav et al. [29] proposed linkage path code for the detection of isomorphism of planar KCs with simple joints and turning pairs. Hwang and Chen [30] proposed an algorithm for the synthesis of all KCs with two inversions.

Cheng and Hsu [31] proposed a method for the synthesis of the kinematic structure of gear KCs with any number of links. Some unconventional methods, such as genetic algorithm and artificial neural network approach as mentioned by the authors [32–33] are also introduced to pursue the issue. Rao [34] discussed about Fuzzy numbers to investigate isomorphism among KCs and inversions. But the main problem with this method is that secondary Fuzzy string is required when the number of links goes on increasing and also calculation becomes so complicated that it is almost impossible to handle the situation. Chang et al. [35] proposed comparison of eigenvalues and eigenvectors of adjacency matrices for isomorphism identification. He et al. [36] proposed a method for the identification of isomorphism. This method was based upon the computation of eigenvectors and eigenvalues of adjacency matrices as well as some permutation operations (some modifications in the existing methods).

Sarkar and Khare [37] proposed the concept of flow path, i.e. flow of motion between links in KCs and mechanisms considering all the paths through which motion can be transmitted from the input to the output link. The authors rejected the concept of shortest path. The idea behind this was that if only the shortest route was considered, the effect of other links that contribute significantly in the transmission of motion from input to output link is completely neglected. Butcher and Hartman [38] proposed an algorithm to enumerate and classify planar simple joints KCs using the hierarchical representation of Fang and Freudenstein.

Cubillo and Wan [39] discussed necessary and sufficient conditions of the eigenvalues and eigenvectors of adjacency matrices of isomorphic chains and also they revised the theory published by Chang et al. [35] about mechanism kinematic chain isomorphism using adjacency matrices. The main advantage of this method is its easy computer execution. Rajesh and Linda [40] established the reliability of the existing spectral techniques i.e. characteristic polynomial and eigenvector approaches for isomorphism detection by determining the number of pairs of non-isomorphic chains, up to 14 links and one, two, and three dof.

Ding and Huang [41] developed a program for the automatic sketching of topological graphs of KCs. They obtained a characteristic adjacency matrix of the canonical perimeter topological graph by labelling vertex coordinates on perimeter topological graphs. After that, they obtained characteristic representation code and this code has been used in the development of the program. Ding and Huang [42] proposed two basic loop operations based on the array representation of loops