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Effects of helix angle and multi-mode on the milling stability prediction using full-discretization method

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ABSTRACT

Regenerative chatter has a negative effect on the quality of machined surfaces in milling and it is thus vital to predict this accurately. The full-discretization method (FDM) is extensively utilized to predict the chatter stability. However, the effects of helix angle and multi-mode usually cause poor prediction by FDM and are often ignored. The existing fourth-order FDM has better computational efficiency and convergence rate than other existing FDMs. In this paper, a fourth-order FDM is optimized to improve prediction accuracy by considering both the helix angle and multi-mode. Based on the numerical stability computation results, it can firstly be observed that the stability lobe diagram (SLD) obtained using the proposed FDM has advantages over the existing SDM in the aspects of both the convergence rate and computational efficiency, and the effect of helix angle on the SLD is determined by both the number of teeth, radial immersion ratio and helix angle of the tool. Furthermore, a detailed investigation on how to detect chatter in milling experiments is made using an integrated method with time-frequency analysis. Finally, comparing the prediction result with the experimental milling results, it can be concluded that the proposed SLD with first three modes excels in prediction accuracy. A quantitative evaluation about the improvement of prediction accuracy is also made. The intention of proposing the updated FDM is to make the chatter prediction more accurate and efficient.

1. Introduction

In industrial machining, precise and efficient milling is a key aim, however, regenerative chatter as a form of self-excited vibration, is a primary obstacle that results in poor quality of machined surfaces and limits metal removal rates. Therefore, predicting regenerative chatter with high precision is important for ensuring a stable milling process and avoiding chatter altogether.

Studies relating to chatter began in the 1960s when Koenigsberger and Tlusty [1] proposed a mechanism of chip regeneration, and a relationship between the maximum cutting depth, dynamic stiffness of the machine tool spindle, and cutting coefficients was suggested. Tobias [2] invented the stability lobe diagram (SLD), which can be used to select the appropriate cutting parameters to avoid chatter during milling. Later, many scholars were devoted to studying the mechanism and prediction methods of chatter, and after decades of development, many prediction methods have been proposed. The above-mentioned methods can roughly be divided into two categories: analytical and numerical.

An analytical method to directly obtain the SLD in the frequency domain was first proposed by Altintas and Budak [3], who analyzed the milling dynamics equation in the frequency domain and presented the zero-order analytical (ZOA) method. The ZOA method is the most efficient method for chatter prediction and has been widely used to predict chatter stability when the radial immersion ratio is not small [4]. However, the prediction accuracy of the ZOA method decreases as the radial immersion ratio becomes smaller. Subsequently, Budak and Altintas [5] optimized the ZOA method and proposed the multi-frequency solution (MFS), which can accurately predict chatter stability, even when the radial immersion ratio is small [6]. Following this, a cutting force model based on the aforementioned analytical methods was presented, which is optimized to include additional effects such as the helix angle of the tool [7,8], process damping [9–11], and so on.

Tang et al. [12] considered that high-order modes might be aroused when the spindle speed or number of teeth increases, and thus the effect of multi-mode would be significant. In order to solve the problem, Tang et al. [12] proposed a method, which was called the lowest envelop method (LEM) [13], to reduce the multi-mode effect on the chatter prediction accuracy. Jin et al. [14] considered the first three modes in the process of acquiring the SLD with the LEM. Later, Wan et al. [13]

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Nomenclature

a	radial immersion of the tool
D	diameter of the tool
$\boldsymbol{E}$	eigenvalues-matrix of transition matrix
$\boldsymbol{E}_r$	eigenvalues-matrix of transition matrix for *r*th-order mode
f_n	natural frequency
f_s	sampling frequency
f_t	feed of per tooth
F_x	cutting force of *x*-axis
$F_{x,jl}$	differential force of x-axis for the layer *l* of flute *j*
F_y	cutting force of *y*-axis
$F_{y,jl}$	differential force of y-axis for the layer *l* of flute *j*
$F_{r,jl}$	differential force of normal direction for the layer *l* of flute *j*
$F_{t,jl}$	differential force of tangential direction for the layer *l* of flute *j*
h_{jl}	chip thickness for the layer *l* of flute *j*
$h_{d,jl}$	dynamical part of differential chip thickness
$h_{s,jl}$	static part of differential chip thickness
K_r	normal cutting coefficient
K_t	tangential cutting coefficient
m	number of time interval per time delay
m_t	modal mass N tooth number of the tool
N_m	number of dominant modes
R	radius of the tool
T	time delay/teeth cutting period
w	axial cutting depth
ω_n	natural angular frequency
ξ	damping ratio
ϕ_{jl}	immersion angle for the layer l of flute j
σ_p	pitch angle
β	helix angle of the tool
Ω	spindle speed
$\boldsymbol{\Phi}$	transition matrix
$\eta_{i,j}\eta_i$	error ratio of prediction accuracy
ϑ_i	improvement ratio of prediction accuracy

optimized the SDM by formulating the dynamic milling process with multiple modes and proved that the updated SDM has higher precision and better adaptability in chatter prediction than the LEM.

Many numerical methods, which acquire the SLD in the time domain, have been proposed in recent years. The difference between the numerical methods mainly lies in the mathematical methods used to analyze the time-delayed differential equations (DDEs) based on a dynamic model. The numerical methods can thus be classified as semi-discretization method (SDM) [15–17], numerical integration method (NIM) [18,19], full-discretization method (FDM) [20–23], Runge-Kutta-based discretization method [24], Euler-Kutta-based discretization method [25], and so on. Among the aforementioned methods, the SDM and FDM have attracted the most interest and have been widely studied. Insperger and Stépán [15] proposed the SDM with the discretization of the delay and periodic terms of the DDEs as the key step in the process, which aimed to realize the numerical computation to obtain the ordinary differential equations (ODEs). The transfer function of the ODEs was thus obtained and the SLD could be acquired according to the Floquet theory [26]. Subsequently, Insperger and Stépán [16], Insperger et al. [17] interpolated the delay and periodic terms of the DDEs with a high-order Lagrange interpolation such that the SDM was more efficient and the convergence rate of the SDM was also improved.

The FDM proposed by Ding et al. [20] was an improvement on the SDM from the perspective of computational efficiency and convergence rate. Besides the delay and periodic terms, the state term of the DDEs was also discretized in the numerical computation process simultaneously. Later, Ding et al. [21], Quo et al. [22], and Ozoegwu et al. [23] interpolated the state term of the DDEs with higher Lagrange interpolation, such that the computational efficiency and convergence rate of the FDM were thereby gradually improved. Ozoegwu et al. [23] found that the computational efficiency and convergence rate of FDM decreased when interpolating the state term to the fifth-order and hence, the fourth-order FDM, which has a fourth-order Lagrange interpolation for the state term of DDEs is regarded as having the best convergence rate and computational efficiency.

Besides the optimization to the discretization process of DDEs above mentioned, further optimization efforts have been made in recent years to improve the prediction accuracy of SDMs and FDMs, by optimizing the dynamical cutting force model and dynamic model. Balachandran and Gilsinn [27], Long and Balachandran [28] took many additional nonlinearities such as the effects of loss-of-contact, multiple regenerative effects, and feed-rate into consideration. Ahmadi and Ismail [29] considered the helix angle of the tool in the modeling process of SDM. Jin et al. [14] included the effect of helix angle in the third-order FDM model and finally, Li et al. [30] considered the helix angle and process damping.

Based on the above literature, it has been known that both the multi-mode effect and helix angle effect have an influence on the prediction accuracy of chatter. In addition, the fourth-order FDM has so far proven to have the best computational efficiency and convergence rate among existing FDMs [23]. Herein, we propose a method with better convergence rate and prediction accuracy based on the existing fourth-order FDM, by taking both the helix angle and multi-mode effects into consideration.

2. Mathematical model of the proposed method

2.1. Assumptions in the model

Besides the effects of helix angle and multi-mode that is mentioned above, the milling process is affected by numerous nonlinearities. In order to simplify the milling process model, a number of assumptions are made such that factors with less influence on the prediction accuracy are neglected as follows:

1) The helix angle and pitch of the tool are fixed and kept constant. Specifically, the value of each tooth's helical angle β and the pitch angle σ_p are assumed to be equal (Fig. 1).
2) The stiffness of the spindle is much less than that of the workpiece, therefore the thick-walled workpiece is clamped by the fixture and the tool is clamped with a large extended length, as illustrated in Fig. 2(a).
3) The impact of process damping, which greatly influences prediction in the low speed range [10,31], is regarded as having little impact on the accuracy of chatter prediction as the high speed range is studied in this paper.
4) The loss-of-contact phenomenon, which easily occurs when the radial immersion ratio is small [28,32], has little influence on chatter prediction as the milling process with large radial immersion ratio is studied in this paper.

2.2. The cutting force model with the effect of helix angle

2.2.1. The formulation of immersion angle

The milling process with a spiral tool is illustrated in Fig. 2(a) where w is the axial cutting depth and Ω is the spindle speed. It can be observed in Fig. 2(b) that the immersion angle $\varphi_{jl}(t)$ changes along the z-axis direction for the existence of helix angle. The jth tooth of the tool is