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Study on the variable coefficient space–time fractional Korteweg de Vries equation

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Abstract In this paper, the fractional Riccati method is modified for solving nonlinear variable coefficients fractional differential equations involving modified Riemann–Liouville derivative. This approach is successfully applied to the variable coefficient space–time fractional Korteweg de Vries (vcSTFKdV) equation. Variety of analytical solutions are obtained. The validity of this approach is discussed. The arbitrariness of the non-integer derivative order α possesses much richer structures. The amplitude increases when the non-integer derivative order increases such that $0.1 < \alpha \leqslant 1$. While it decreases the non-integer derivative order increases such that $0 < \alpha < 0.1$. From the graphical presentation of the obtained solutions it is observed that changing the non-integer derivative order value α affects the soliton behavior in a fundamental way; therefore, the non-integer derivative order can modify the wave shape without changing the properties of the medium, the nonlinearity and the dispersive effects.

1. Introduction

Fractional differential equations (FDEs) are generalizations of classical differential equations (DEs) of integer order to non-integer one. Even though this subject was initiated in the first half of the 19th century it becomes a hot topic of research nowadays. Recent studies show that the FDEs are more appropriate models than their counterparts of integer order when realizing complex nonlinear phenomena [1–37]. The derivatives of non-integer order depend on the local behavior of the function or accumulate the whole information of the function which is known as the memory effect. As a result of this, abundant scientific fields have been used FDEs such as physics, biology, chemistry, mathematics, engineering, communication, diffusion process, porous media, power-law non-locality and power-law long-term memory [12–37]. Meanwhile, the development of new studies to solve FDEs has attracted considerable much attention and numerous techniques have been put forward. Examples include Adomian decomposition, variational iteration,

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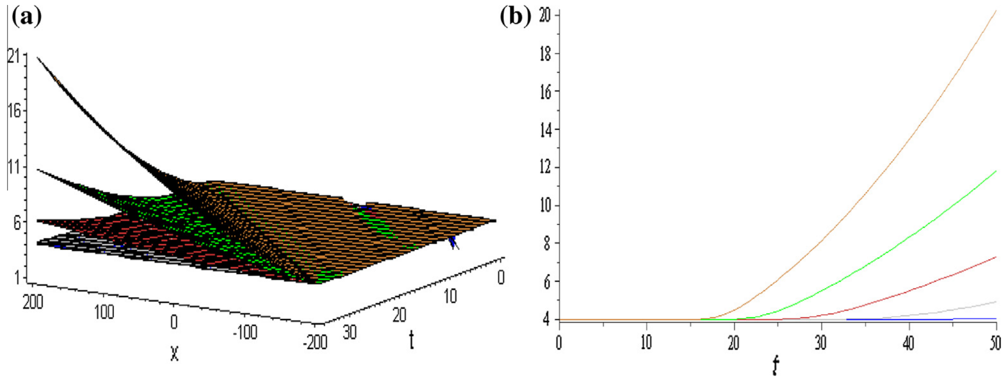

Figure 1 Evolutional behavior of u_3 given by Eq. (18) with $a_0 = a_1 = k = \tau(t) = 1$ (a) u_3 versus x and t. (b) u_3 versus t at $x = 10$ where $\alpha = 0.6, 0.7, 0.8, 0.9, 1$.

homotopy perturbation, differential transform, Lie symmetry group, finite difference, finite element, Laplace transform, Fourier transform, exponential function, fractional sub-equation, (G'/G)-expansion method, first integral method, operational method, fractional Riccati, improved fractional Riccati and fractional projective Riccati [10–37]. Based on these methods, several FDEs have been studied. One of the important approaches to obtain analytical solutions of FDE is the fractional sub-equation method. The basic idea of this method is the assumption that the exact solution of FDEs can be expressed as a polynomial of a function which satisfies simple and solvable equation, namely sub-equation or auxiliary equation. Some of the used sub-equations are the fractional Riccati equation, fractional elliptic equation and fractional projective Riccati equation. In this work, we modify the fractional Riccati (FR) method to treat FDEs with variable coefficients. The vcSTFKdV is solved and discussed.

This work is organized as follows: mathematical background and definition of the modified Riemann–Liouville (mRL) derivative are given in Section 2. The modification of FR method to treat variable coefficients FDEs is presented in Section 3. The vcSTFKdV equation is studied in Section 4. Finally, we conclude the paper.

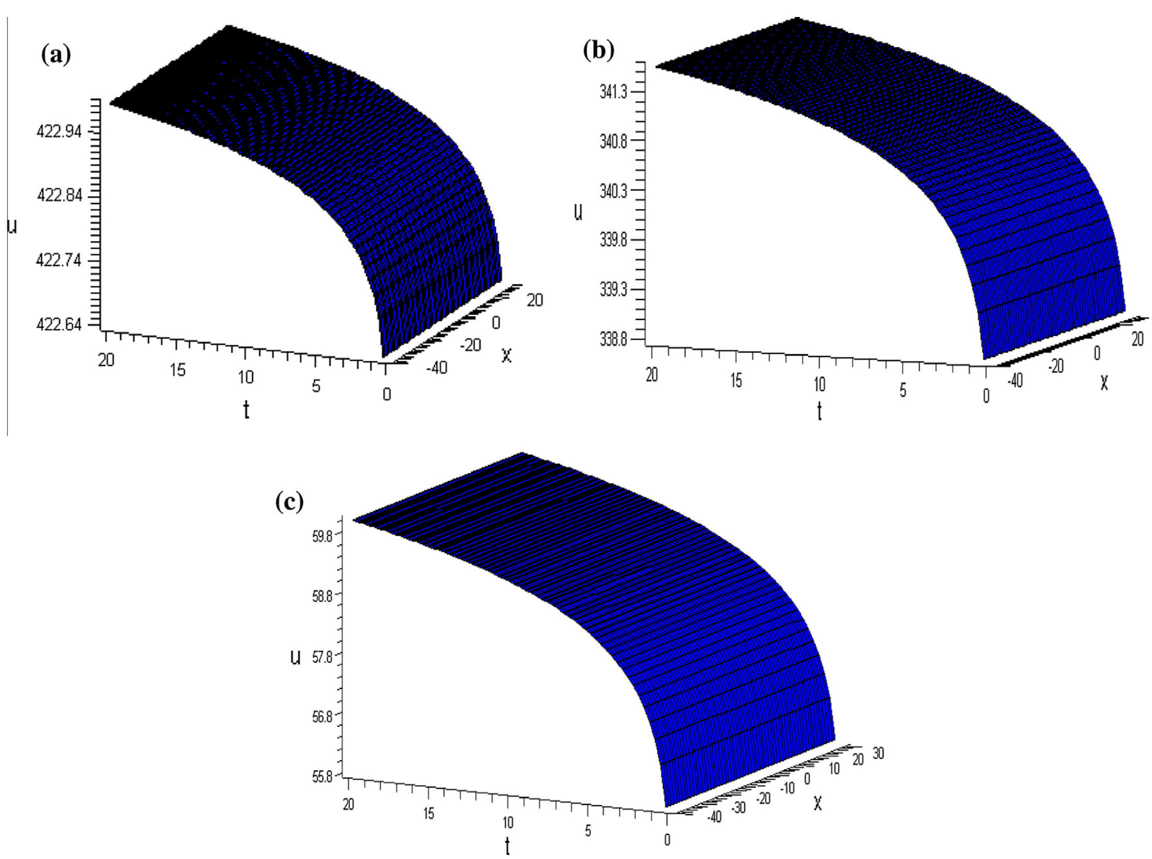

Figure 2 Evolutional behavior of u_3 versus x and t with $a_0 = a_1 = k = \tau(t) = 1$ at (a) $\alpha = 0.0001$, (b) $\alpha = 0.001$, (c) $\alpha = 0.01$.

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