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Jet noise modelling and control/Modélisation et contrôle du bruit de jet

# Global mode-based control of laminar and turbulent high-speed jets ☆

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ABSTRACT

The emitted noise from round jets is reduced using linear feedback controllers designed using structural sensitivity analysis. Linear global modes inform the selection and placement of the controller, and Navier–Stokes simulations are used to demonstrate effectiveness in a Mach-1.5 cold axisymmetric jet and in a Mach-0.9 cold turbulent jet. In both jets, each fitted with a cylindrical nozzle, the control reduces the radiated noise and modifies the baseflow in a way that enhances the relative amplitudes of low-frequency $St \approx 0.05$ global modes that do not have significant support in the acoustic field.

## 1. Introduction

Several decades of experimental, theoretical, and computational-based investigations have suggested that the basic structure of the loudest jet noise sound sources, i.e. those at low frequencies and radiating towards the downstream jet axis, are consistent with a wavepacket structure [1]. Different approaches have been used to describe these wavepackets, such as linear stability theory applied to spatially-developing shear layers and jets [2–9]. In jets, these wavepackets are intermittent, advecting disturbances that are correlated over distances that exceed the integral scales of turbulence [1]. Educing these structures from numerical or experimental data has, for example, included projecting the data onto locally-parallel instability waves [10] or parabolized stability equations (PSE, Gudmundsson and Colonius [11]), a process that can be augmented by using a proper orthogonal decomposition (POD) as a filter [12]. Numerically predicting the noise from the wavepackets modeled as instability waves has also been investigated by, for example, Balakumar [13], Yen and Messersmith [14], and Cheung et al. [15].

Global mode analysis complements these approaches in that it provides a direct means of analyzing the behavior of linear disturbances whose superposition can replicate a wavepacket character. Theofilis [16] and Theofilis and Colonius [17] review the use of global modes while Jordan and Colonius [1] review wavepackets as a model for low-frequency jet noise. Although they are more computationally intensive to compute than PSE modes, global modes are more faithful to the full

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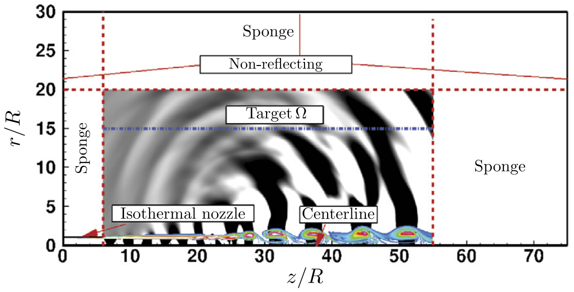
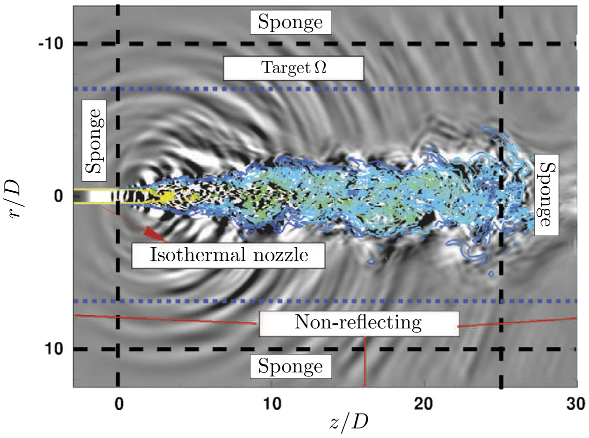


(a) Axisymmetric Mach-1.5 jet.



(b) Turbulent Mach-0.9 jet.

**Fig. 1.** Domain and boundary condition specifications for axisymmetric and turbulent jet calculations.

governing equations in that they do not invoke any spatial variation approximations of the flow field and, more importantly, allow for upstream–downstream coupling within the jet.

Several investigators examined the use of global modes to describe jet hydrodynamic and acoustic fields. Nichols and Lele [18], for example, used a global mode analysis about a parameterized baseflow to obtain the optimal transient response of a cold, supersonic jet and identified upstream-propagating acoustic modes that were not resolved in the earlier PSE analyses. Garnaud et al. [19] investigated the linear stability dynamics of incompressible and compressible isothermal jets by means of their optimal initial perturbations and of their global eigenmodes; see also Coenen et al. [20]. More recent investigations have focused on a resolvent analysis and determining optimal input–output gains for time-periodic disturbances (e.g., Semeraro et al. [21]).

In this paper we use global modes to control high-speed round jets using linear feedback. We include the nozzle in the simulations and analysis because it sets inflow conditions and couples flow disturbances. Moreover, the nozzle is the most natural platform for control of noise. We use the same numerical formulation to compute the adjoint global modes, which provide sensitivity via the wavemaker approach [22,23], and utilize it to develop the control. Adjoint global modes are key to assessing the importance of individual global modes in seeking quieter jets. Our control uses a structured sensitivity-based approach originally developed in Bodony and Natarajan [24] and subsequently expanded in Natarajan et al. [25].

We focus exclusively on the axisymmetric ($m = 0$) global modes because they are an important component of high-Reynolds-number turbulent jets [26], and thus an important testbed for control development. Jet noise control using higher order modes may be approached using the control method we develop without modification. We demonstrate the control first on an axisymmetric Mach-1.5 cold jet, followed by a turbulent Mach-0.9 cold jet that is axisymmetric in the mean.

The governing equations, numerical methods, boundary conditions, and the formulation and solution to the eigenvalue problems are discussed in Section 2. The results (simulation details, global mode analysis, and control) for the axisymmetric Mach-1.5 jet are given in Section 3. Section 4 gives the results corresponding to the cold Mach-0.9 turbulent jet. Following a discussion in Section 5, conclusions are drawn in Section 6.

## 2. Governing equations, boundary conditions, and numerical methods

The jets considered are shown in Fig. 1. The equations of mass, momentum, and energy conservation are solved for a compressible, viscous fluid with an ideal gas equation of state ($p = \rho R T$), Fourier law of heat conduction ($\boldsymbol{q} = -k\nabla T$) and Newtonian viscous stresses $\tau = \mu[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\mathsf{T}}] + \lambda(\nabla \cdot \boldsymbol{u})\mathsf{I}$, where $\mathsf{I}$ is the identity tensor and $\boldsymbol{u}$ is the velocity field. For the axisymmetric jet the equations are expressed in cylindrical polar coordinates $\boldsymbol{x} = (r, \theta, z)$ using cylindrical polar velocities

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