Optik - International Journal for Light and Electron Optics 176 (2019) 198–201

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Conservation laws for optical solitons with anti-cubic and generalized anti-cubic nonlinearities

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ARTICLE INFO

OCIS codes:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Solitons
Conservation laws
Anti-cubic nonlinearity
Generalized anti-cubic nonlinearity

ABSTRACT

This paper lists the conservation laws for optical solitons that are studied with anti-cubic nonlinearity and its generalized counterpart. The laws are given in terms of Gauss' hypergeometric functions.

1. Introduction

Conservation laws is the pillar of strength for advancement in the field of nonlinear sciences. Without a knowledge of these con-laws, it is not possible to retrieve additional knowledge of any physical system or any optoelectronic phenomena, in this case. Therefore, it is imperative to discuss these laws in order to venture further into optoelectronics as will be discussed. This paper will list the conservation laws for optical solitons that come with anti-cubic (AC) nonlinearity and its generalized version that is known as generalized AC nonlinearity [1–10]. This AC nonlinearity first appeared during 2003 and later this form of nonlinear optical fiber gained popularity and flooded several journals with a variety of novel results for this law [6]. Recently, the governing nonlinear Schrödinger's equation (NLSE) with AC law is generalized and its soliton solutions have been located [4]. This paper will list the con-laws for both of these fibers. It must be however noted that earlier con-laws for AC nonlinearity have been enumerated. However, they were computed with a different form of bright solitons [10]. The details are sketched in the next couple of sections.

2. Anti-cubic nonlinearity

The NLSE with AC form of nonlinearity is written as:

$$\mathrm{i}q_t + \mathrm{a}q_{xx} + \left(\frac{b_1}{|q|^4} + b_2\,|q|^2 + b_3\,|q|^4\right)q = 0 \tag{1}$$

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https://doi.org/10.1016/j.ijleo.2018.09.074
Received 12 July 2018; Accepted 16 September 2018

In (1), the complex-valued function $q(x;t)$ represents the soliton profile. The independent variables x and t are the spatial and temporal co-ordinates respectively. The first term represents temporal evolution of the soliton pulses where $i = \sqrt{-1}$ and a is the coefficient of group velocity dispersion (GVD). The last three terms stem from nonlinearity that are together known as AC non-linearity. If $b_1 = 0$, one recovers NLSE with cubic-quintic nonlinear fibers. Solitons solutions are the outcome of a delicate balance that persists between GVD and nonlinearity. For (1), the 1-soliton solution, as obtained by extended trial function method is written as [3,5]:

$$q(x, t) = \frac{A}{\sqrt{D + \cosh[B(x - \mathrm{vt})]}} e^{i(-\kappa x + \omega t + \theta_0)} \tag{2}$$

Here, A is the amplitude of the soliton and B is its inverse width, together with v being the soliton velocity and finally D is a free external parameter. From its phase portion, κ is the soliton frequency and ω being its wave number with θ_0 as the phase constant. The three conserved quantities are soliton power (P), linear momentum (M) and Hamiltonian (H) that are respectively given by [10]:

$$P = \int_{-\infty}^{\infty} |q|^2 \mathrm{dx} \tag{3}$$

$$M = \mathrm{ia} \int_{-\infty}^{\infty} (q^* q_x - \mathrm{qq}_x^*) \mathrm{dx} \tag{4}$$

and

$$H = \int_{-\infty}^{\infty} \left(a\, |q_x|^2 + \frac{b_1}{|q|^2} - \frac{b_2}{2}\, |q|^4 - \frac{b_3}{3}\, |q|^6 \right) \mathrm{dx} \tag{5}$$

2.1. Conservation laws

For solitons given by (2), these conserved quantities are:

$$P = \int_{-\infty}^{\infty} |q|^2 \mathrm{dx} = \int_{-\infty}^{\infty} \frac{A^2}{D + \cosh\tau} \mathrm{dx} = \frac{2A^2}{B} F\left(1, 1; \frac{3}{2}; \frac{1 - D}{2}\right) \tag{6}$$

$$M = \mathrm{ia} \int_{-\infty}^{\infty} (q^* q_x - \mathrm{qq}_x^*) \mathrm{dx} = 2a\kappa \int_{-\infty}^{\infty} \frac{A^2}{D + \cosh\tau} \mathrm{dx} = \frac{4a\kappa A^2}{B} F\left(1, 1; \frac{3}{2}; \frac{1 - D}{2}\right) \tag{7}$$

and

$$\begin{aligned} H &= \int_{-\infty}^{\infty} \left(a\, |q_x|^2 + \tfrac{b_1}{|q|^2} - \tfrac{b_2}{2}\, |q|^4 - \tfrac{b_3}{3}\, |q|^6 \right) \mathrm{dx} \\ &= \int_{-\infty}^{\infty} \left\{ \mathrm{aA}^2 B^2 \tfrac{\sinh^2\tau}{(D + \cosh\tau)^3} + \tfrac{a\kappa^2 A^2}{D + \cosh\tau} + \tfrac{b_1}{A^2}(D + \cosh\tau) - \tfrac{b_2}{2} \tfrac{A^4}{(D + \cosh\tau)^2} - \tfrac{b_3}{3} \tfrac{A^6}{(D + \cosh\tau)^3} \right\} \mathrm{dx} \\ &= \tfrac{2}{3} \mathrm{aA}^2 \mathrm{B}\ \mathrm{F}\left(3, 1; \tfrac{5}{2}; \tfrac{1 - D}{2}\right) + \tfrac{2a\kappa^2 A^2}{B} F\left(1, 1; \tfrac{3}{2}; \tfrac{1 - D}{2}\right) + \tfrac{b_1}{A^2} \int_{-\infty}^{\infty} (D + \cosh\tau) \mathrm{dx} \\ &\quad - \tfrac{b_2 A^4}{3B} F\left(2, 2; \tfrac{5}{2}; \tfrac{1 - D}{2}\right) - \tfrac{4 b_3 A^6}{45B} F\left(3, 3; \tfrac{7}{2}; \tfrac{1 - D}{2}\right) \end{aligned} \tag{8}$$

where in (6)–(8), the notation $\tau = B(x - vt)$ has been introduced. Here, Gauss' hypergeometric function is listed as:

$$F(\alpha, \beta; \gamma; z) = \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n}{(\gamma)_n} \frac{z^n}{n!} \tag{9}$$

and the Pochhammer symbol is:

$$(p)_n = \begin{cases} 1 & n = 0, \\ p(p + 1)\cdots(p + n - 1) & n > 0. \end{cases} \tag{10}$$

The condition for convergence with hypergeometric function is

$$|z| < 1. \tag{11}$$

which, for (31)–(33), leads to

$$-1 < D < 3. \tag{12}$$

Furthermore, Rabbe's test of convergence implies

$$\gamma < \alpha + \beta, \tag{13}$$

which is valid for M_j for $1 \le j \le 3$.

It needs to be noted that the Hamiltonian for optical solitons with AC nonlinearity does not exist because of the presence of a divergent integral that is given by the coefficient of b_1. Thus, there are only two conserved quantities for AC nonlinearity and these are soliton power and its linear momentum.