Economics Letters 173 (2018) 10–14

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

A generalised stochastic volatility in mean VAR[☆]

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HIGHLIGHTS

- Introduce a stochastic volatility in mean VAR with new features.
- Model allows for correlation in the shocks to level and volatility.
- Model allows the data to dynamically affect the volatilities.
- Paper provides a Gibbs sampling algorithm for estimation.

ARTICLE INFO

Article history:
Received 24 May 2018
Received in revised form 30 August 2018
Accepted 31 August 2018
Available online xxxx

JEL classification:
C3
C11
E3

Keywords:
VAR
Stochastic volatility in mean
Error covariance

ABSTRACT

This note introduces a VAR with stochastic volatility in mean where the shocks of the volatility equations and the observation equations are allowed to be correlated. We provide a Gibbs algorithm to approximate the posterior distribution and demonstrate the proposed methods by estimating the impact of financial uncertainty shocks on the US economy.

1. Introduction

A large number of recent empirical papers that aim to measure the effect of uncertainty shocks have employed vector autoregressions with stochastic volatility in mean (VARSVOL). As the estimation of these models is complex, some simplifying assumptions are usually adopted. It is typically assumed that shocks to the stochastic volatility equations are independent of shocks to the endogenous variables. This assumption is not necessarily innocuous as many economic shocks can affect both the level and the conditional variance of macroeconomic variables.

This note describes the estimation of a VARSVOL where the shocks of the transition equations are allowed to be correlated with those of the observation equation. In econometric terms, allowing for such a correlation implies that the model has a structure akin to a reduced form VAR where the structural shocks are identified in a second step. This allows the researcher to distinguish amongst uncertainty and level shocks by using SVAR techniques rather than imposing exogeneity of the former a priori.

We provide an MCMC algorithm to approximate the posterior distribution of the parameters in this extended model and provide an application where financial uncertainty shocks are estimated via a small VAR for the US.

The paper is organised as follows: The model is described in Section 2 with the estimation algorithm summarised in Section 2.1. Section 3 presents the empirical results while Section 4 concludes.

2. Empirical model

We consider the following state–space model:

$$\tilde{h}_t = \alpha + \theta \tilde{h}_{t-1} + \sum_{j=1}^{Q} d_j Z_{t-j} + S^{1/2}\eta_t \tag{1}$$

$$Z_t = c + \sum_{j=1}^{P} \beta_j Z_{t-j} + \sum_{k=1}^{K} b_k \tilde{h}_{t-k} + H_t^{1/2} e_t \tag{2}$$

where Z_t is a $N \times 1$ vector of endogenous variables.

The stochastic volatilities are denoted by the $N \times 1$ vector $\tilde{h}_t = [h_{1t}, h_{2t}, ..h_{N,t}]'$ and $H_t = diag\left(\exp\left(\tilde{h}_t\right)\right)$. In Eq. (1), θ and d_j denote the $N \times N$ coefficient matrices, while α is an $N \times 1$ intercept

☆ This paper benefited from comments by an anonymous referee and the editor Badi H. Baltagi.

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https://doi.org/10.1016/j.econlet.2018.08.044

vector. The shocks to the transition equation (1) have a variance $S = diag(\tilde{s})$ with $\underbrace{\tilde{s}}_{N\times 1} = [s_1, s_2, \ldots, s_N]'$. Note that θ can be a non-diagonal matrix with the elements of $\tilde{h}_t$ allowed to have a dynamic relationship amongst themselves. The observation equation of the system is the VAR model in Eq. (2) where β_j and b_k are coefficient matrices of dimension $N \times N$ with c denoting an $N \times 1$ vector of intercepts. The equation implies that $\tilde{h}_t$ is allowed to have a lagged impact on the endogenous variables.

The $M = 2N$ disturbances $\varepsilon_t = \begin{pmatrix} \underbrace{\eta_t}_{N\times 1} \\ \underbrace{e_t}_{N\times 1} \end{pmatrix}$ are distributed normally $N(0, \Sigma)$ where the diagonal elements of Σ are restricted to equal 1:

$$\underbrace{\Sigma}_{M\times M} = \begin{pmatrix} \Sigma_\eta & \Sigma'_{\eta_t e_t} \\ \Sigma_{\eta_t e_t} & \Sigma_{e_t} \end{pmatrix} \tag{3}$$

The time-varying covariance matrix of the reduced form residuals of the system in Eqs. (1) and (2) can be written as:

$$\underbrace{\Omega_t}_{M\times M} = \begin{pmatrix} S^{1/2} & 0 \\ 0 & H_t^{1/2} \end{pmatrix} \begin{pmatrix} \Sigma_\eta & \Sigma'_{\eta_t e_t} \\ \Sigma_{\eta_t e_t} & \Sigma_{e_t} \end{pmatrix} \times \begin{pmatrix} S^{1/2} & 0 \\ 0 & H_t^{1/2} \end{pmatrix}' \tag{4}$$

Thus the model allows for correlation between the shocks to the level of the endogenous variables and volatilities.

The main difference between the model proposed here and stochastic volatility in mean and VARSVOL models used in recent papers such as Koopman and Uspensky (2002), and Mumtaz and Surico (2018) is that the covariance between level shocks and those to second moments is allowed to be non-zero. This implies that in order to identify structural shocks u_t from the reduced form disturbances in the system, additional assumptions are required. In particular, the structural shocks can be estimated as $u_t = A_{0,t}^{-1}\varepsilon_t$ where $A_{0,t}A'_{0,t} = \Omega_t$. The contemporaneous impact matrix $A_{0,t}$ could be obtained using one of the techniques developed in the large literature on structural VARs (SVAR).[1]

2.1. Gibbs sampling algorithm

We approximate the marginal posterior distribution of the parameters and states $B, S, \Sigma, \tilde{h}_t$ using a Gibbs sampling algorithm. A sketch of the algorithm is provided here with implementation details of each step given in the technical appendix. The algorithm samples from the following conditional posterior distributions:

1. $G\left(B|S, \Sigma, \tilde{h}_t\right)$. The conditional posterior distribution of the coefficients $B = vec\left([\alpha, \theta, d_1, \ldots, d_Q, c, \beta_1, \ldots, \beta_P, b_1, \ldots, b_K]\right)$ can be obtained by writing Eqs. (1) and (2) as a seemingly unrelated regression system. Given $\tilde{h}_t$, the disturbances of the system are normal with covariance matrix Σ. With a normal prior, the conditional posterior of B is also normal. The Kalman filter can be used find the mean and the variance of the conditional posterior taking into account the time-variation in H_t.
2. $G\left(S|B, \Sigma, \tilde{h}_t\right)$. The correlation amongst the disturbances of the transition equation $\tilde{\eta}_t = S^{1/2}\eta_t$ implies that the conditional posterior for the elements of S is non-standard and a Metropolis step is required. A candidate density that displays satisfactory performance in simulations is the inverse Gamma (IG) distribution centred at the posterior moments calculated under the assumption that $\tilde{\eta}_t$ are uncorrelated, i.e. $IG(v_1, T_1)$, $v_1 = \tilde{\eta}'_{it}\tilde{\eta}_{it} + v_0$ and $T_1 = T_0 + T$ where v_0, T_0 denote prior moments and T is the sample size. In practice, this can also be combined with a IG distribution centred on the previous draw to obtain a mixture proposal density. Note that given B, Σ, $\tilde{h}_t$ and a draw of S from the candidate density, the likelihood can be easily calculated with the process described in the appendix.
3. $G\left(\Sigma|B, \tilde{h}_t, S\right)$. Given B and the variances $S, \tilde{h}_t$, the residuals ε_t. The draw of the restricted covariance matrix is obtained via the independence Metropolis algorithm described in Chan and Jeliazkov (2009).
4. $G\left(\tilde{h}_t|\Sigma, B, S\right)$. The observation equation of the state–space system can be written as:

$$Z_t - H_t^{1/2}\mu_{e_t|\eta_t} = c + \sum_{j=1}^{P}\beta_j Z_{t-j} + \sum_{k=1}^{K} b_k\tilde{h}_{t-k} + \tilde{e}_t$$
$$var\left(\tilde{e}_t\right) = \Omega_t = H_t^{1/2}\Sigma_{e_t|\eta_t}H_t^{1/2\prime}$$

where $\mu_{e_t|\eta_t}$ denotes the conditional mean of e_t and $\Sigma_{e_t|\eta_t}$ is the conditional variance:

$$\mu_{e_t|\eta_t} = \eta_t\Sigma_{\eta_t}^{-1}\Sigma'_{\eta_t e_t}$$
$$\Sigma_{e_t|\eta_t} = \Sigma_{e_t} - \Sigma_{\eta_t e_t}\Sigma_{\eta_t}^{-1}\Sigma'_{\eta_t e_t}$$

We treat η_t as a state variable in this step and write the transition equation as

$$F_t = C + \Psi F_{t-1} + N_t$$

where $F_t = \begin{pmatrix} \eta_{t+1} \\ \eta_t \\ \tilde{h}_t \\ . \\ \tilde{h}_{t-k} \end{pmatrix}$. Note that the residual of the transformed observation equation $\tilde{e}_t$ is uncorrelated with N_t. As described in the appendix, we employ a particle Gibbs step (see Andrieu et al. (2010) and Lindsten et al. (2014)) to sample F_t from its conditional posterior distribution. The use of particle Gibbs to draw the state vector implies that a linearisation of the observation equation (as in Omori et al. (2007)) is not required. Moreover, given the large number of parameters in the proposed model, the Gibbs algorithm used here is likely to be more efficient than a maximum likelihood or a particle Metropolis Hastings approach that operates on all parameters simultaneously.

We conduct a small Monte Carlo experiment to evaluate the performance of the algorithm. We generate data from the following DGP

$$\begin{pmatrix} \ln h_{1t} \\ \ln h_{2t} \end{pmatrix} = \begin{pmatrix} 0.85 & -0.1 \\ 0.1 & 0.85 \end{pmatrix}\begin{pmatrix} \ln h_{1t-1} \\ \ln h_{2t-1} \end{pmatrix} + \begin{pmatrix} -0.05 & 0.01 \\ -0.05 & 0.01 \end{pmatrix}\begin{pmatrix} Y_{t-1} \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} s_{11}^{1/2}e_{1t} \\ s_{22}^{1/2}e_{2t} \end{pmatrix}$$

$$\begin{pmatrix} Y_t \\ X_t \end{pmatrix} = \begin{pmatrix} 0.3 \\ -0.3 \end{pmatrix} + \begin{pmatrix} 0.5 & -0.1 \\ 0.1 & 0.5 \end{pmatrix}\begin{pmatrix} Y_{t-1} \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} -0.05 & 0.01 \\ -0.05 & 0.01 \end{pmatrix}\begin{pmatrix} \ln h_{1t-1} \\ \ln h_{2t-1} \end{pmatrix} + \begin{pmatrix} h_{1t}^{1/2}e_{3t} \\ h_{2t}^{1/2}e_{4t} \end{pmatrix},$$

[1] In other words, the model proposed above is a multi-variate extension of stochastic volatility models with leverage considered in Jacquier et al. (2004), Omori et al. (2007) and Pitt et al. (2014).