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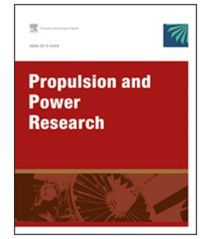

ORIGINAL ARTICLE

Unsteady time-dependent incompressible Newtonian fluid flow between two parallel plates by homotopy analysis method (HAM), homotopy perturbation method (HPM) and collocation method (CM)

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KEYWORDS

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Parallel porous plates;
Unsteady flow

Abstract Analytical and numerical analyses have performed to study the problem of the flow of incompressible Newtonian fluid between two parallel plates approaching or receding from each other symmetrically. The Navier–Stokes equations have been transformed into an ordinary differential equation using a similarity transformation. The powerful analytical methods called collocation method (CM), the homotopy perturbation method (HPM), and the homotopy analysis method (HAM) have been used to solve nonlinear differential equations. It has been attempted to show the capabilities and wide-range applications of the proposed methods in comparison with a type of numerical analysis as fourth-order Runge–Kutta numerical method in solving this problem. Also, velocity fields have been computed and shown graphically for various values of physical parameters. The objective of the present work is to investigate the effect of Reynolds number and suction or injection characteristic parameter on the velocity field.

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Nomenclature

t	time
c	a constant related to the inlet condition
K	strength of the suction or injection
p	pressure
Δp_n	pressure drop in the normal direction
Re	Reynolds number
R	residual
u, v	velocity components along x, y axes, respectively
V_w	injection velocity
$\dot{a}(t)$	the distance between two plates

Greek letters

υ	kinematic viscosity
ρ	fluid density

1. Introduction

The problem of unsteady time-dependent flow between parallel plates has many crucial applications in science and technology. Among them are hydrodynamic lubrication, aerodynamic heating, polymer technology, petroleum industry and biomechanics. Many researchers have investigated such flows with different geometries and different flow conditions [1–5]. The flow between parallel plates in presence of magnetic field also attracted many researchers. A large class of fluid flow phenomena is described by the governing equations of motion, see for instance [6–9].

In recent decades, many attempts have been made to develop analytical methods for solving such nonlinear equations. One of them is the perturbation method [10], which is strongly dependent on a so called small parameter to be defined according to the physics of the problem. Since these equations cannot be solved via the conventional analytical techniques, recent attempts have been focused on constructing an analytical solution for these equations using the advanced developed methods such as adomian's decomposition method (ADM) [11–13], homotopy perturbation method (HPM) [14–19], variational iteration method (VIM) [20,21], differential transformation method (DTM) [22–24], collocation method (CM) [25,26], homotopy analysis method (HAM) [27–34], optimal homotopy analysis method (HAM) [35,36] and least square method (LSM) [37,38]. Liao introduced the basic idea of homotopy in topology to propose a general analytical method for nonlinear problems, namely the homotopy analysis method, which does not need any small parameter. This method has been successfully applied to solve many types of nonlinear problems.

The aim of this study is to investigate, the effect of physical parameters Reynolds number and suction or injection characteristic parameter on flow behavior of unsteady incompressible fluid flow between two parallel plates.

2. Problem statement and mathematical formulation

We consider an incompressible two dimensional flow of Newtonian fluid between two parallel infinite rectangular plates in Cartesian coordinates. These two plates are placed a distance $a(t)$ at apart from each other and t denotes time. Figure 1 shows the schematic diagram and the coordinate system for the considered flow.

We also consider that the upper plate which is at $y = a(t)$ is moving toward the lower plate with velocity $\dot{y} = \dot{a}(t)$ and the lower porous plate which is at $y = 0$ is fixed.

The equations of motion of this flow are [33,34,39]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

where ρ denotes density, P denotes pressure and ν denotes kinematic viscosity. For boundary conditions we have:

$$y = a(t) \rightarrow u(x,y,t) = 0, \; v(x,y,t) = \dot{a}(t) \tag{4}$$

$$y = 0 \rightarrow u(x,y,t) = 0, \; v(x,y,t) = K\dot{a}(t) \tag{5}$$

As we can see for the axial velocity, we have no slip boundary condition. Also, as the distance between the two plates varies with $\dot{a}(t)$ and the lower plate is stationary we can write for the upper plate that $v = \dot{a}(t)$. For the lower plate we insert a constant parameter like K which is a candidate for the strength of the suction or injection. Here $K > 0$ corresponds to suction and $K < 0$ corresponds to injection. The transformations (for detail see Refs. [40,41],

$$u = \frac{C - x}{a(t)}\dot{a}(t)f'(\eta), \; v = \dot{a}(t)f(\eta), \tag{6}$$

where $\eta = \frac{y}{a(t)}$.

Here C is a constant related to the inlet condition of the channel. Using the transformations (6) in Eqs. (1)–(3), continuity equation will be satisfied automatically and the Navier–Stokes equations reduce to:

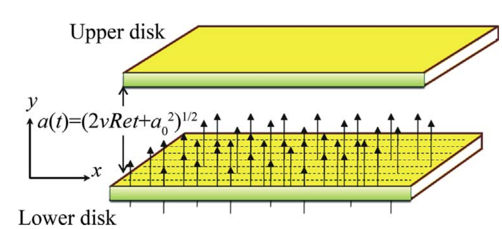

Figure 1 Schematic diagram and the coordinate system for the considered flow.

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