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# Min–Max exact and heuristic policies for a two-echelon supply chain with inventory and transportation procurement decisions

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ABSTRACT

We study the problem in which one supplier delivers a product to a set of retailers over time by using an outsourced fleet of vehicles. Since the probability distribution of the demand is not known, we provide a Min–Max approach to find robust policies. We show that the optimal Min-Expected Value policy can be very poor in the worst case. We provide a Min–Max Dynamic Programming formulation that allows us to exactly solve the problem in small instances. Finally, we implement a Min–Max Matheuristic to solve benchmark instances and show that it is very effective.



## 1. Introduction

A supply chain can be represented by a multi-echelon system, i.e. a system having several stages like procurement, manufacturing or transportation of items. In a multi-echelon approach, the main goal is to find the optimal inventory decisions by considering all the stages in the supply chain at once (Klosterhalfen and Minner, 2010). The scientific literature is rich of contributions devoted to the study of real-world supply chains. There are several studies that deal with the optimal allocation of the safety stocks in the supply chain by taking into account demand uncertainty. The two main methods are the so-called *Stochastic-Service Model* (SSM) and the *Guaranteed-Service Model* (GSM), which were introduced by Clark and Scarf (1960) and Simpson and Kenneth (1958), respectively. Eruguz et al. (2016) present a survey of these methods for the multi-echelon inventory optimization. We refer the reader to it for an in depth analysis of multi-echelon supply chain systems.

*Supply Chain Management* requires to make decisions that optimize the amount of resources involved in several sequential phases of the production–distribution process. In the *Vendor Managed Inventory* (*VMI*) systems, the aim of the supplier is to optimize the inventory levels of several retailers to meet the demand of one or several products, while minimizing the total cost, given by the sum of inventory and transportation costs. It is well known that inventory and transportation costs conflict between them: Solutions with high transportation cost but low inventory cost are obtained by delivering the products frequently, while the contrary happens delivering the products rarely over time. Recently, all problems in which the sum of inventory and transportation costs is minimized over time are referred to as *Inventory Routing Problems* (*IRP*), even if the transportation cost is not strictly the cost of the routes, but it is for example a fixed transportation cost. We refer to

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Campbell et al. (1998) and to Bertazzi et al. (2008) for an introduction to IRP, to Bertazzi and Speranza (2012) for a tutorial on problems with decisions over time only (no routing), to Bertazzi and Speranza (2013) for a tutorial on IRP (with routing) and to Andersson et al. (2010) and Coelho et al. (2013) for surveys on IRP. In the case in which the transportation cost is the routing cost and the demand is deterministic, finding an optimal solution of the corresponding *IRP* consists in solving a well known NP-hard optimization problem (see Archetti et al., 2007 for a benchmark IRP model and for the first branch-and-cut algorithm for IRP, Desaulniers et al. (2015) for an innovative mathematical formulation and the state-of-the-art branch-price-and-cut algorithm for solving it, Cordeau et al. (2015) for an IRP with several different products, Niakan and Rahimi (2015) for a multi-objective IRP).

In the case of stochastic demand, i.e. in the case in which a probability distribution of the demand is given for each retailer, finding an optimal policy consists in solving a very complex dynamic programming model, dealing with the three well known *curses of dimensionality* (huge cardinality of the state space, huge cardinality of the control space, prohibitive computation of the expected cost) (see Powell, 2011). More precisely, the total expected cost is given by the sum of four different cost components: (1) the inventory cost at the supplier, (2) the inventory cost at the retailers, (3) the penalty cost arising whenever a stock-out occurs at the retailers, and (4) the transportation cost to serve the retailers. In this case, referred to as the *Stochastic Inventory Routing Problem*, the word *policy* is used instead of *solution*, to mean the rule followed by the supplier to supply each retailer in each state of the system (inventory level at the supplier and inventory level of each retailer). The aim is to find a policy that minimizes the total expected cost over the planning horizon (Min-Expected Value policy). We refer to Bertazzi et al. (2013) for exact and heuristic algorithms for this problem, to Huang and Lin (2010) for the first Ant Colony Optimization for IRP with stochastic demand and to Javid and Azad (2010) for the integration of the IRP with stochastic demand with location, allocation and capacity decisions.

A different case takes place when a third actor is used to perform the deliveries to the retailers, i.e. transportation procurement is used. This happens whenever the supplier has not its own fleet of vehicles. Transportation procurement can greatly reduce operating costs. An application is when transportation services are bought via Third Party Logistics (3PL) marketplaces that allow to pay the lowest transportation cost. Another application is when companies outsource last-mile deliveries of small quantities to carriers working as freight contractors in a regional district. We refer to Jothi Basu et al. (2015) for a recent survey on full truckload transportation procurement and to Bertazzi et al. (2015) for the first *Stochastic Inventory Routing Problem with Transportation Procurement*. This problem is an IRP with stochastic demand in which a fixed cost is paid at each time period to buy a transportation capacity.

This paper gives a significant contribution to the literature in the areas of Transportation and Logistics. We study a new problem, the robust version of the *Stochastic Inventory Routing Problem with Transportation Procurement*. This problem is defined on the same two-echelon supply chain with inventory and transportation procurement decisions, but demand is now uncertain. This means that the demand is not known and the probability distribution of the demand is not given as well. This problem is new as the previous contributions in literature are focused either on the stochastic version of this problem (see Bertazzi et al., 2013) or on the classical Inventory Routing Problem with stochastic demand (see Bertazzi et al., 2015) or on different robust approaches (see Solyali et al., 2012). For the first time, a Min–Max exact and matheuristic approach is applied to a two-echelon supply chain with inventory and transportation procurement decisions to find robust policies, i.e. policies that minimize the maximum cost. We refer to Ben-Tal et al. (2009) for a recent overview of the state of the art in Robust optimization. We focus on the approach proposed in Bertsekas (2005) and Shapiro (2011), which is based on the Min–Max Exact Dynamic Programming algorithm. Our aim is to design exact and heuristic Min–Max policies, i.e. policies able to protect against the worst-case realization of the demand of the retailers, and to compare them with the classical Min-Expected Value policies obtained by minimizing the expected cost. More precisely, our contributions are the following. We intuitively know that, when the aim of the decision-maker is to minimize the maximum cost, a Min-Expected Value policy typically provides a lower level of protection than a Min–Max policy and therefore an increase in the cost due to stock-out penalty costs. For the first time, we provide analytical results to formally prove the gap we have in the worst case by comparing the optimal Min-Expected Value policy with the optimal Min–Max policy. Since this gap is very large, the decision maker should be very careful in applying the classical Min-Expected Value policy. This implies that *ad hoc* solution methods should be designed for the robust version of the problem. Then, we consider the opposite approach. Suppose that decision-maker applies a Min–Max policy in a problem where the expected cost is minimized. Again, intuitively we know that the performance of this policy can be poor. As before, our contribution is to provide analytical results to formally prove the gap we have in the worst case by comparing the optimal Min–Max policy with the optimal Min-Expected Value policy. Since this gap is very large, the Min–Max objective should be really well justified to be adopted. Our third contribution is to formulate a dynamic programming model of the Min–Max problem. This allows us to apply an Exact Dynamic Programming algorithm (different than the classical one, given that we look for an optimal min–max policy) to computationally compare the optimal Min–Max policy with the classical optimal Min-Expected Value policy. Since even this algorithm is affected by the three curses of dimensionality related to the exponential state space, the exponential control space and the exponential number of realizations of the demand, our fourth contribution is to design and implement for the first time a Min–Max Matheuristic algorithm, able to find near-optimal policies in benchmark instances. Finally, we provide a lower bound on the optimal Min–Max cost to evaluate the performance of the matheuristic.

Note that, although the paper is focused on finding optimal and heuristic Min–Max policies for the case in which all realizations of the demand are considered (i.e. the most conservative robust approach), the Exact Min–Max Dynamic