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Braess Paradox of traffic networks with mixed equilibrium behaviors

Wei (Walker) Wang [a], David Z.W. Wang [a,*], Huijun Sun [b], Zengzhe Feng [b], Jianjun Wu [c]

[a] School of Civil and Environmental Engineering, Nanyang Technological University, Singapore 639798, Singapore
[b] Key Laboratory for Urban Transportation Complex Systems Theory and Technology of Ministry of Education, Beijing Jiaotong University, Beijing, China
[c] State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing, China

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ABSTRACT

Under the user equilibrium (UE) behavior assumption, the Braess Paradox (BP) and its variations have been well investigated. However, users do not always follow the UE behavior. In reality, there are likely quiet a few non-collaborative Cournot–Nash (CN) players coexisting with UE players in the common traffic network. Users in a CN player are completely collaborative to minimize their total travel cost and users subordinating to different players are perfectly competitive. Considering both UE and CN players in the congested network, it remains unclear that under what conditions the BP will occur. In this paper, the BP occurrence conditions under the UE–CN mixed equilibrium are firstly investigated using the classical Braess network with linear link cost function. Then, the BP conditions are studied to the ordinary grid network with nonlinear link cost function. It is shown that the BP occurrence in the conventional Braess network depends upon the link travel time function parameters and the demand level of users controlled by UE/CN players, and the BP occurs in the grid network only for certain demand combinations of users under one UE player and two CN players.

1. Introduction

The counterintuitive phenomenon that building new roads or enlarging capacities of existing roads in a traffic network might increase the total network cost is called Braess Paradox (BP). The reason for the BP is that individual traveler always acts selfishly when making route choice decisions (Braess, 1968, 2005; Murchland, 1970; Pas and Principio, 1997; Szeto, 2011). BP and its variations have been investigated extensively. The studies on BP mainly focuses on two aspects.

One was on the network features with general flow-dependent link travel time functions. Frank (1981) and Steinberg and Zangwill (1983) investigated the occurrence of BP in the traffic networks but just considered linear link congestion functions. Dafermos and Nagurney (1984) detected whether the BP happens by means of a positive semi-definite matrix with a consideration of asymmetric link travel cost function. Pas and Principio (1997) discussed the existence of BP in the symmetrical traffic network structure. Valiant and Roughgarden (2006) proved that the BP would occur easily in a large random network. Zverovich and Avineri (2015) took into account the non-symmetric network configuration and showed that the BP is inevitable when the total travel demand falls within a determinate range of values.

The other aspect was on users' route choice behavior in the network. Zhao et al. (2014) investigated the BP under stochastic user equilibrium (SUE) behavior assumption. Di et al. (2014) put forward the general existence condition of the BP based on the boundedly rational user equilibrium (BRUE). Hwang and Cho (2015) proposed a generalized inverse approach to

* Corresponding author.
E-mail address: wangzhiwei@ntu.edu.sg (D.Z.W. Wang).

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detect the BP with a non-unique equilibrium route flow solution. Szeto and Jiang (2014) discussed a Braess-like paradox in transit assignment, where providing new lines to a transit network or increasing the frequency of an existing line may not improve the expected total system travel cost. Nagurney et al. (2007) developed an evolutionary variational inequality to study a time-dependent BP. Zhang et al. (2008a) analyzed the BP in the dynamic traffic assignment context. Lin and Lo (2009) solved the BP problem with the consideration of dynamic user equilibrium (DUE).

In the above literatures, the BP and its aforementioned variants have been well investigated in traffic networks with different topology structures under UE, SUE, BRUE or DUE behavior assumptions. However, in fact, users do not always follow the above principles, especially when there are some self-optimizing oligopolistic Cournot firms in the transportation network. Each firm is referred to as a Cournot–Nash or CN player. As is well known, users belong to the shared CN player collaborate with each other to minimize their total cost and users in different players compete against others (Haurie and Marcotte, 1985). Such CN players may be large transport enterprises, shipping companies, or a group of car users subscribing route guidance information. For instance, advanced transportation information system provides route guidance information to a group of car users. The system sets route guidance information and disseminate it to the users so that the travel time cost of the entire group of users is minimized, which could be regarded as a CN player. Harker (1988) firstly proposed a UE–CN mixed equilibrium assignment model, where a slice of users are commanded by various CN players while others behave in a UE manner. Bennett (1993) presented an equative mathematical programming model for the UE–CN mixed traffic assignment problem. Yang et al. (2007) extended the study of Harker (1988) and Bennett (1993) and took into account a global system optimum player in the UE–CN mixed traffic assignment model. Yang and Zhang (2008) and Zhang et al. (2008b) investigated the existence of anonymous/uniform road pricings which could decentralize the system optimum flow pattern into the UE–CN mixed behavior equilibrium. Yu and Huang (2010) studied the upper bounds of the efficiency loss for the UE–CN mixed traffic assignment model. He et al. (2013) proposed tradable credit schemes, which is superior to optimal link tolls, in traffic networks considering the UE–CN mixed equilibrium behavior.

Given the UE–CN mixed behavioral assumption, how UE and CN players respond to the new added links in a traffic network has never been investigated in previous studies. Moreover, it is unclear that under what conditions the BP occurs in a traffic network when there exist both UE and CN players. This paper aims to fill in this research gap by exploring when the BP occurs under the mixed equilibrium from the conventional Braess network to the ordinary grid network. Specifically, the paper analyzes the impact of the demand level of UE/CN players on the occurrence of the BP. This study shows that, considering both UE and CN players in a shared traffic network, BP occurs only if the combined demand level of each group of players lies within a determinate range of values. The results of this paper can thus assist the transportation network planners in understanding how to avoid the occurrence of BP when network design plans are proposed considering both UE and CN players in the network behaviors.

This paper is structured as follows. The brief review of UE–CN mixed equilibrium is given in Section 2. In Section 3, the BP analysis performs with the classical Braess network. In Section 4, the BP analysis is discussed in the grid network. Section 5 concludes this paper.

2. A brief review of UE–CN mixed equilibrium

Consider a traffic network $G = (N, A)$, where N and A are the set of nodes and links, respectively. Suppose that for every origin–destination (O–D) pair in the traffic network, all of the users are subject to either a specific CN player or a UE player. The notations used throughout this paper are summarized as follows:

U	the set of UE player
K	the set of CN players
R^U	the set of O–D pair for which users behave UE principles
R^k	the set of O–D pair for which users are commanded by CN player $k \in K$
R^K	$R^K \equiv \bigcup_{k \in K} R^k$
R	$R \equiv R^U \cup R^K$
d_r	fixed travel demand between O–D pair $r \in R$
P_r	the set of paths between O–D pair $r \in R$
f_p	flow on path $p \in P_r,\ r \in R$
δ_{ap}	1 if path $p \in P$ includes link $a \in A$, and 0 otherwise
x_a^U	flow on link a from the set R^U
$\mathbf{x}^U$	$\mathbf{x}^U \equiv (\cdots, x_{a-1}^U, x_a^U, x_{a+1}^U, \cdots)$
x_a^k	flow of link a from the set R^k, $k \in K$
$\mathbf{x}^k$	$\mathbf{x}^k \equiv (\cdots, x_{a-1}^k, x_a^k, x_{a+1}^k, \cdots)$
$\mathbf{x}^K$	$\mathbf{x}^K \equiv (\cdots, \mathbf{x}^{k-1}, \mathbf{x}^k, \mathbf{x}^{k+1}, \cdots)$
x_a^K	$x_a^K \equiv \sum_{k \in K} x_a^k$
x_a	$x_a \equiv x_a^U + x_a^K$
$\mathbf{x}$	$\mathbf{x} \equiv (\cdots, x_{a-1}, x_a, x_{a+1}, \cdots)$
$t_a(x_a)$	travel time on link $a \in A$