Transportation Research Part E 93 (2016) 453–473

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

The competitive facility location problem under disruption risks

Ying Zhang [a,b], Lawrence V. Snyder [c], Ted K. Ralphs [c], Zhaojie Xue [d,*]

[a] *Zhejiang Cainiao Supply Chain Management Co., Ltd., Hangzhou 310000, China*
[b] *Department of Industrial Engineering, Tsinghua University, Beijing 100084, China*
[c] *Department of Industrial and Systems Engineering, Lehigh University, 200 West Packer Ave., Mohler Lab, Bethlehem, PA 18015, USA*
[d] *Department of Transportation Engineering, College of Civil Engineering, Shenzhen University, Shenzhen 518060, China*

ARTICLE INFO

Article history:
Received 6 January 2016
Received in revised form 3 June 2016
Accepted 5 July 2016
Available online 25 July 2016

Keywords:
Competitive location
Facility disruptions
Bilevel optimization
Local search
Variable neighborhood search

ABSTRACT

Two players sequentially locate a fixed number of facilities, competing to capture market share. Facilities face disruption risks, and each customer patronizes the nearest operational facility, regardless of who operates it. The problem therefore combines competitive location and location with disruptions. This combination has been absent from the literature. We model the problem as a Stackelberg game in which the leader locates facilities first, followed by the follower, and formulate the leader's decision problem as a bilevel optimization problem. A variable neighborhood decomposition search heuristic which includes variable fixing and cut generation is developed. Computational results suggest that high quality solutions can be found quickly. Interesting managerial insights are drawn.

1. Introduction

This paper introduces the *competitive facility location problem under disruption risks* (CFLPD), a discrete facility location model that, to the best of our knowledge, is the first to incorporate possible disruptions into a competitive facility location problem. In many industries, *service competition* is present among multiple firms such as supermarkets or gas stations. Customers may choose among competing facilities based on distance (as we assume in this paper), quality, brand loyalty, or other factors. In addition, facilities may face *disruptions* from time to time due to natural disasters, labor actions, or power outages. When a facility is disrupted, its customers may seek service from another operational facility belonging to the same player; they may seek service from a facility belonging to a different player; or their sales may be lost entirely. In either of last two cases, the customer's original service provider loses revenue, and in all three cases, the customers incur higher service costs. For express carriers such as FedEx and UPS, both service competition and delivery delays or labor disputes will influence their brand recognition, service quality and market share. For example, a FedEx store may lose its customers if it delivers packages late due to labor actions or other disruptions, or if a UPS store is nearby. This highlights the need for an optimal facility deployment that considers both service competition and probabilistic facility disruptions.

We consider a supplier–receiver network with multiple customers and two noncooperative firms (the players of the Stackelberg game): the leader and the follower. The players make facility location decisions sequentially, with each aiming to maximize its own market share or revenue. This setup is well modeled as a Stackelberg game (Dempe, 2002) in which each player has exactly one move. The leader will first open B facilities, anticipating that the follower will react rationally

* Corresponding author.
E-mail addresses: yingzhang8996@gmail.com (Y. Zhang), larry.snyder@lehigh.edu (L.V. Snyder), ted@lehigh.edu (T.K. Ralphs), zjxue@szu.edu.cn (Z. Xue).

http://dx.doi.org/10.1016/j.tre.2016.07.002
1366-5545/

by optimally placing K facilities. Each customer has a demand and seeks the nearest operating facility for service. We assume a binary preference model in which each customer chooses only a single operating facility for service at any one time.

In addition, the facilities opened are subject to disruptions. When a facility is disrupted, it cannot serve customers. Following Snyder and Daskin (2005), we assume that customers of disrupted facilities are reassigned to another (functional) facility. In particular, we assign each customer to multiple facilities in a sequence of *assignment levels* $r = 1, 2, \cdots$. [1] For each customer, the closest facility ($r = 1$), the so-called the primary facility, will serve it under normal circumstances. If the primary facility fails, the customer is served by its first backup facility ($r = 2$). If that facility fails too, it is served by its level-3 facility, and so on. In general, a facility assigned to a customer at level r serves that customer if all $r - 1$ facilities at lower levels have failed. If a customer's primary facility fails and the nearest operating facility is owned by the other player, then the player that owns the primary facility will lose that customer until the disruption ends. If all of the facilities assigned to the customer are disrupted, the customer is lost by both of the players.

We formulate the problem as a binary bilevel linear optimization problem (BBLP). The model determines the optimal locations for the leader in order to maximize her market share, under the strongest possible response by the follower. If the facilities are assumed to be always reliable, i.e., each customer can always be captured by the nearest facility, then we obtain as a special case the discrete $(r|p)$-centroid problem (RPCP) (Alekseeva et al., 2010) and the closely related competitive maximal covering location model (Serra and ReVelle, 1994; Seyhan et al., 2015). (The competitive maximal covering model assumes that customers will only patronize facilities within a given coverage radius, whereas the RPCP allows any assignment, regardless of distance; otherwise, the two problems are identical.)

It has been shown that the discrete RPCP is NP-hard; in fact, it belongs to the class of $\sum_2^P$-hard problems (Noltemeier et al., 2007). This means that to check whether a (leader's) decision is feasible requires solving an NP-hard problem (to optimize the follower's strategy). This study addresses this complicated but also realistic problem. Our main contributions are as follows: First, we construct a BBLP model for this new type of facility location problem. Second, we develop a matheuristic based on variable neighborhood decomposition search (VNDS) which includes variable fixing and cut generation interactively. We further show that this matheuristic can be extended to a large class of BBLP directly. Third, extensive experiments and sensitive analysis demonstrate the effectiveness of our approach. Results on RPCP benchmarks show that the VNDS matheuristic is very promising compared to the current best heuristics and exact approaches for this special case. Results on the more general CFLPD instances draw many interesting managerial insights on the approximate facility deployment strategies and market share competition.

The remainder of this paper is organized as follows. We review the relevant literature in Section 2 and formulate the CFLPD as a BBLP in Section 3. A matheuristic using VNDS is provided in Section 4. The numerical experiment design and computational results are presented in Section 5. Finally, Section 6 concludes the paper and discusses future research.

2. Literature review

Facility location problems have been extensively studied in the past few decades, due to the wide variety of applications that arise in placing distribution centers, warehouses, gas stations, and fire stations, as well as in constructing communication networks, and so on. Two of the most well-studied problems are the p-median problem and the maximal covering location problem. The p-median problem is to locate p facilities so that the total demand-weighted distance between each customer and the nearest facility is minimized. The maximal covering location problem seeks to locate a fixed number (e.g., p) of facilities so that the number of covered demands is maximized. For overviews of these two classical models, see, for example, Snyder (2010) or Daskin (2013).

Both the p-median problem and the maximal covering location problem ignore the effect of competition on the location decision and assume that there is a single decision-maker. However, many firms face location-based competition, and failing to account for this competition when choosing facility locations can result in lower than anticipated market share. *Competitive facility location problems* take this competition into account; most assume there are two players who successively open their facilities, each aiming to capture customers and maximize revenue. For reviews of competitive location models, see Eiselt et al. (1993), Serra et al. (1994), Eiselt and Laporte (1997), Kress and Pesch (2012) and Farahani et al. (2014).

One competitive location problem, the RPCP, has attracted increased attention in the past five years. In this problem, the leader places p facilities on a graph knowing that the follower will react by placing r facilities.[2] The goal of both the leader and the follower is to maximize its own market share (Alekseeva et al., 2010). This problem is often formulated as a BBLP. Recent algorithms have been tested using instances with up to 100 customers, 100 potential facilities and $p = r = 20$ or 30 from the Discrete Location Problems benchmark library.[3] Exact approaches—including the iterative exact method by Alekseeva et al. (2010), the branch-and-cut (RP-B&C) method by Roboredo and Pessoa (2013) and the modified iterative exact method (MEM) by Alekseeva and Kochetov (2013)—guarantee global optimality but are very computationally intensive (e.g., more than 10 h for $p = r = 10$; see Roboredo and Pessoa (2013)). The iterative exact method by Alekseeva et al. (2010) is based on a

[1] Note that we index the levels beginning at $r = 1$, whereas Snyder and Daskin (2005) and others index them beginning at $r = 0$.

[2] When discussing the RPCP, we will use p and r to denote the number of facilities opened by the leader and the follower, respectively, instead of B and K as in our model, to remain consistent with the existing research.

[3] http://www.math.nsc.ru/AP/benchmarks/english.html.