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A multi-start heuristic approach for the split-delivery vehicle routing problem with minimum delivery amounts

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ABSTRACT

We propose a new multi-start solution approach for the split-delivery vehicle routing problem with minimum delivery amounts (SDVRP-MDA). Initial solutions are generated by both node-insertion and route-addition procedures with a single parameter to control the restart. These solutions are then improved by a variable neighborhood descent metaheuristic with a novel search operator inspired by node-ejection chains. We test the proposed approach with 32 benchmark instances for four different minimum delivery fractions. Using the proposed algorithm, out of 128 cases tested, we find 81 best known solutions and 34 new best solutions; overall, we find 43 new best solutions.

1. Introduction

This paper addresses the split-delivery vehicle routing problem with minimum delivery amounts (SDVRP-MDA), in which the demand of a customer can be split and delivered to the customer by several vehicles on different routes, but the customers each require a minimum delivery amount (MDA) every time they are visited. The SDVRP-MDA, to which attention was first drawn by Gulczynski et al. (2010), is a restrained version of the split-delivery vehicle routing problem (SDVRP) introduced earlier by Dror and Trudeau (1989). The SDVRP and the SDVRP-MDA are both variants of the conventional vehicle routing problem (VRP).

By allowing split deliveries without MDA requirements, the distance traveled and the number of vehicles employed can be potentially reduced by up to 50% (Archetti et al., 2006a, 2008a). Split deliveries may thus result in significant cost savings for in-house logistics operations. However, when shipped to outside customers, split deliveries incur extra work and create distractions for those customers who are visited more than once. These distractions are undesirable to customers, unless the amount or value of each delivery reaches a certain level of satisfaction for the customer. Gulczynski et al. (2010) thus proposed the SDVRP-MDA with an additional MDA constraint to the SDVRP.

In practice, the MDA can be imposed as a monetary value by the distributor. For example, according to Gulczynski et al. (2010), Pizza Hut in the USA delivered orders placed by phone or online if the order was at least $8.00 (approximately). Similarly, the biggest chain store 7-Eleven in Taiwan (7-ELEVEN Taiwan, 2015) and Domino's Pizza in Singapore (Domino's Pizza Singapore, 2015) will do the same if the order is at least NT$300 (approximately $9.10) and SG$15 (approximately $10.73), respectively. Moreover, Everhealth Pharmaceuticals (Everhealth Pharmaceuticals, 2015) in Taiwan, and Tesco Groceries in

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the UK (Tesco PLC, 2015) will deliver orders which are at least NT$2,000 (approximately $60.62) and £40 (approximately $61.30), respectively.

Gulczynski et al. (2010) also proposed a minimum delivery fraction p ($0 \leqslant p \leqslant 1$) to determine the MDA for each customer i, i.e., $MDA_i = \lceil p \times d_i \rceil$ where d_i is the demand of customer i. In this study, we set p in the domain of [0, 0.5], i.e., $0 \leqslant p \leqslant 0.5$. This is because, when $p > 0.5$, it is impossible to have two deliveries each with a fraction of more than half, and the problem thus reduces to a conventional VRP. Accordingly, the VRP and the SDVRP can be regarded as special cases of the SDVRP-MDA with $p > 0.5$ and $p = 0$, respectively. Fig. 1 illustrates an example of a simple network with three customers, where the edge labels are distances, the node labels in parentheses are demands (or delivery amounts), and the vehicle capacity is 100. For $p > 0.5$, the problem is equivalent to a VRP. In its optimal solution (Fig. 1a), there are three routes without split deliveries, and the total distance traveled is 48. For $p = 0$, the problem is equivalent to a SDVRP; and its optimal solution (Fig. 1b) shows that customer 2 (denoted by two nodes in gray) receives two deliveries from two separate routes. In this SDVRP solution, the total distance traveled (i.e., 38), is less than that in the VRP solution (i.e., 48). In Fig. 1(c), the optimal solution for the SDVRP-MDA (with $p = 0.2$) contains two routes, and the total distance traveled (i.e., 43) is longer than that in the SDVRP solution (i.e., 38). Due to $p = 0.2$, each customer must have at least 20% of its demand delivered on a single route. As shown in Fig. 1(c), customer 3 receives two deliveries, and the smaller amount of these two is 30, which meets the MDA requirement (i.e., $30 > MDA_3 = 19 = 95 \times 0.2$). On the other hand, the solution in Fig. 1(b) is infeasible for $p = 0.2$, because the delivery amount on one of the two routes for customer 2 is only 5, which is less than MDA_2 (i.e., $7 = 35 \times 0.2$).

Similar to the SDVRP that was proved to be *NP*-hard (Dror and Trudeau, 1990), the SDVRP-MDA is also *NP*-hard. Despite the abundant literature on the SDVRP, the existing literature on the SDVRP-MDA is limited. To the best of our knowledge, the only method available for solving the SDVRP-MDA in the literature is the hybrid heuristic method proposed by Gulczynski et al. (2010). In this hybrid method, conventional VRP solutions were generated first using a modified savings algorithm (Yellow, 1970). These initial VRP solutions were then improved by an endpoint mixed integer program with minimum delivery amounts (EMIP-MDA) to provide the first solutions for the SDVRP-MDA. These solutions were further improved by an enhanced record-to-record travel algorithm (ERTR) developed by Groër et al. (2011). Overall, the method was called EMIP-MDA + ERTR. Gulczynski et al. (2010) also proposed a new set of test problems for the SDVRP-MDA by modifying the 21-instance set introduced by Chen et al. (2007) for the SDVRP. These instances were applied to test EMIP-MDA + ERTR for four different p values (i.e., $p = 0.1$, 0.2, 0.3, and 0.4) with good results. Recently, Xiong et al. (2013) proved that, in general, the maximal saving in travel cost for the SDVRP-MDA is the same as that for the SDVRP (i.e., as much as 50%). The only exception is for a special case when $p = 0.5$. In such a case, the potential saving is as much as one third.

In this paper, we propose a new multi-start two-phased variable neighborhood descent heuristic approach for solving the SDVRP-MDA. This multi-start approach is implemented with a parameter α ($0 \leqslant \alpha \leqslant 1$). For each α in a restart, initial solutions are first generated by a split-delivery route construction algorithm, which we call SRC. These initial solutions are then improved by a variable neighborhood descent (VND) procedure. Overall, the proposed approach is referred to as SRC + VND, which incorporates the following unique features for the SDVRP-MDA: (1) two different yet complementary constructive procedures to enhance the diversification of the search, (2) an adaptive procedure to construct MDA-restrained split-delivery routes using a mix of node insertion and route addition steps, and (3) a novel neighborhood search operator inspired by node-ejection chains to enhance the intensification of the search. We have tested our solution algorithms with all available benchmark problems of the SDVRP-MDA. Out of the 32 instance problems each tested with four different p values (i.e., $p = 0.1$, 0.2, 0.3, and 0.4), we have obtained 81 existing best solutions and 34 new ones from the proposed SRC + VND algorithm. Moreover, the proposed algorithm relies on only one parameter and requires minimal tuning efforts.

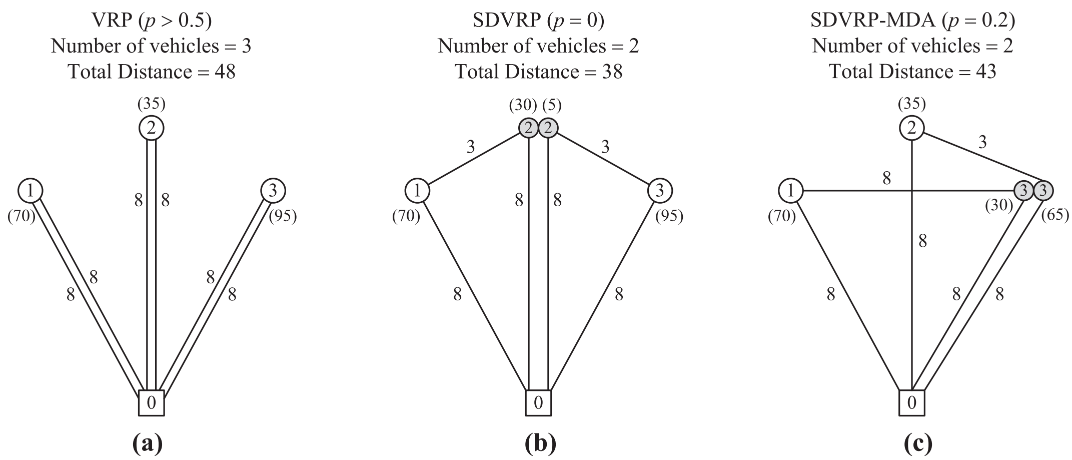

Fig. 1. Comparison of the VRP, SDVRP and SDVRP-MDA. Nodes in gray denote split deliveries.