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Branch-and-price algorithm for the location-routing problem with time windows

Sattrawut Ponboon *, Ali Gul Qureshi, Eiichi Taniguchi

Department of Urban Management, Kyoto University, Kyotodaigaku-Katsura, Nishikyo-ku, Kyoto 615-8540, Japan

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A B S T R A C T

This study proposes a branch-and-price algorithm to solve the Location-Routing Problem with Time Windows (LRPTW) which has never been attempted with the exact solutions before. The problem is solved by the simplex algorithm in the master problem and elementary shortest path problems with resource constraint corresponding to column generation in the subproblem until only the non-negative reduced cost columns remain. The proposed algorithm can solve many testing instances effectively. The computational results and the effect of time windows are also compared and discussed.

1. Introduction

In the modern distribution management, integrated logistics systems to optimize a product support and customer service have become a primary necessity. Determining a suitable location of the business center is considered one of the most essential steps in supply chain management, i.e. setting up factory, warehouse, retail store, or public services, i.e. hospital, police station, fire station, etc. (Hassanzadeh et al., 2009). Thus, the facility location is required at several points throughout the supply chain.

Traditionally, the facility location and vehicle routing has been determined and carried out at the different levels. While the routing can be solved more frequently at the short term operational stage, the facility must be located earlier in the long term strategic planning. Some argue that an integration of these two problems is impractical since they are in the different planning framework, which makes it inappropriate to calculate them together. Nonetheless, it was proved that the combination of Location-Routing Problem (LRP) reduces the cost over the long-term horizon. Solving them together, early in the planning horizon provides benefits and positive impacts for both operators and society (Salhi and Nagy, 1999; Marinakis, 2009; Hemmelmayr et al., 2012; Escobar et al., 2013; Contardo et al., 2014; Escobar et al., 2014a).

Just in time scheduling has become a key element in the modern distribution management. Since the customers often request the service time and deadlines, the future LRP studies should be extended to consider the presence of time windows (Min et al., 1998). By adding the complexity of time window constraint into the problem, a proper and effective method must be developed to deal with a combination of facility location and routing simultaneously which is complex by its NP-hard (Dror, 1994). Despite its importance, from our best knowledge of literature, all of the LRP studies that considered time windows were based on heuristic approaches, none of them attempted at the optimal solution using the exact algorithm.

* Corresponding author. Tel.: +81 75 383 3231; fax: +81 75 950 3800.
E-mail addresses: sattrawut.ponboon.47n@st.kyoto-u.ac.jp (S. Ponboon), aligul@kiban.kuciv.kyoto-u.ac.jp (A.G. Qureshi), taniguchi@kiban.kuciv.kyoto-u.ac.jp (E. Taniguchi).

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This study, therefore, proposes the integrated approach to determine the exact solution of the Location-Routing Problem with Time Windows (LRPTW) using branch-and-price algorithm.

The paper is structured as follows. Section 2 reviews the literature on the LRP, time window, and exact solution approaches and Section 3 summarizes the contribution of this paper. Section 4 provides the general formulation of the LRPTW with the explanation of used variables. Section 5 describes the branch-and-price algorithm comprised of a set-partitioning or column generation problem, the pricing problem, the branching schemes, and accelerating processes. Section 6 presents the testing instances, the computational results, and key information of the LRPTW using branch-and-price algorithm and conclusions are provided in Section 7 at the end of the paper.

2. Literature review

2.1. Location-routing problem

There are two types of distribution to be considered; direct trip and tour trip. The direct trip requires the vehicle to visit only one customer and return to the base as a fire service. On the other hand, the tour trip requires the vehicle to visit more than one customer before returning to the base such as a postman (Hassanzadeh et al., 2009). If the service is a direct trip, it is considered a location–allocation problem. If the service is a tour trip, it is referred as the LRP (Fig. 1).

The location problem does not require a routing determination similar to the Hamiltonian *p*-median problems. Likewise, the multi-depot vehicle routing problem does not solve the facility location since it is implicitly determined (Vidal et al., 2012; Cordeau and Maischberger, 2012; Subramanian et al., 2013; Escobar et al., 2014b). However, in the LRP, two decisions are considered interdependently, which facilities should be opened and which vehicle routes should be built. The former is based on ordinary location problem while the latter is developed with the fundamentals of the Vehicle-Routing Problem (VRP) seeking to serve a set of customers with a fleet of vehicles with minimum distribution cost.

In the early works, the difficulty of computational process limited the LRP to be presented only as the conceptual framework (Von Boventer, 1961; Maranzana, 1964; Webb, 1968; Lawrence and Pengilly, 1969; Christofides and Eilon, 1969; Higgins, 1972). Watson-Gandy and Dohrn (1973) were the first authors who considered the vehicle routing together with depot locating using a non-linear profit function. To evaluate the benefit of this integration, Salhi and Rand (1989) assessed the effect of ignoring routing when locating the depots and proved that the best solution in locating stage does not guarantee the best solution in routing stage. The early survey studies show that the LRP did not receive much attention thereafter for a long time (Balakrishman et al., 1987; Laporte, 1988, 1989; Berman et al., 1995; Min et al., 1998). However, according to the latest comprehensive surveys (Nagy and Salhi, 2007; Drexl and Schneider, 2013; Prodhon and Prins, 2014), the LRP-related research are growing in number.

Similar to the classical vehicle-routing problem related research, there are number of variants in the LRP researches. Nagy and Salhi (2007) classified the LRP-related research using nine (09) variants including hierarchical structure, type of input data, planning period, solution method, objective function, solution space, number of depots, number and types of vehicles, and route structure. On the basis of that information, this study uses standard LRP, which can be defined with following characteristics: deterministic, single-period, exact solution, cost minimization, discrete location, multiple depots, homogeneous fleet, and *depot-customer-depot* structure.

2.2. Time windows

In the context of logistics management, much attention has been paid in improving the service quality by increasing customer satisfaction. Some might argue that the delivery time is relatively small compared to the time horizon of the facility

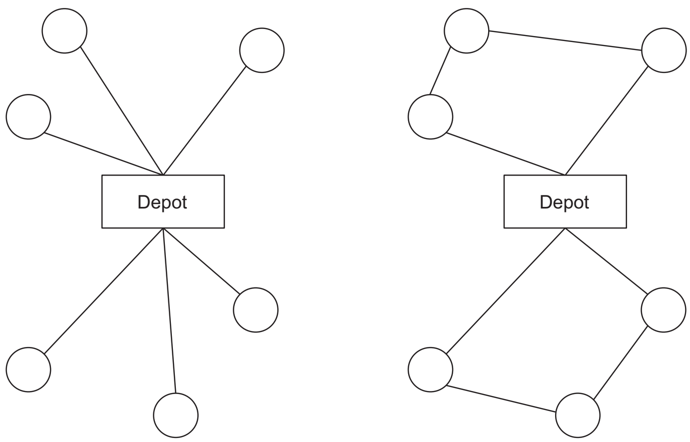

Fig. 1. Location–allocation problem (left) and location-routing problem (right) (Daskin, 1995).