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A novel discrete network design problem formulation and its global optimization solution algorithm

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ABSTRACT

Conventional discrete transportation network design problem deals with the optimal decision on new link addition, assuming the capacity of each candidate link addition is predetermined and fixed. In this paper, we address a novel yet general discrete network design problem formulation that aims to determine the optimal new link addition and their optimal capacities simultaneously, which answers the questions on whether a new link should be added or not, and if added, what should be the optimal link capacity. A global optimization method employing linearization, outer approximation and range reduction techniques is developed to solve the formulated model.

1. Introduction

The discrete network design problem (DNDP) involves the optimal decision on addition of new links or roadway segments to an existing transportation network, subject to a limited investment budget. Traditionally, given a group of candidate links with fixed capacities, the DNDP is formulated as 0–1 decision problem aiming to determine the optimal road construction plan. The objective of DNDP is to optimize transportation network performance while considering the drivers' routing behavior, for example, following deterministic user equilibrium (DUE) (Sheffi, 1985). The DNDP is typically formulated as a bi-level program with the upper-level minimizing the total travel time cost and the lower-level describing the equilibrium flow pattern.

The DNDP has been widely investigated in previous research works, and it is widely recognized as one of the most difficult frontiers in transportation study due to its computational difficulties in solving the mixed-integer nonlinear nonconvex, bi-level program formulation. Yang and Bell (1998) reviewed a number of models and solution algorithms for network design problem (NDP) based on bi-level programming. Magnanti and Wong (1984) presented a unifying framework for deriving a bunch of algorithms for DNDP and reviewed some computational experience in solving NDP. LeBlanc (1975) proposed a branch-and-bound (B&B) algorithm for solving the upper-level problem of DNDP. Poorzahedy and Turnguist (1982) adopted a well-behaved function to substitute the original total user cost objective function and formulated a single-level model. A B&B based heuristic algorithm was also given in their research. By applying the concept of support function to express the relationship between improvement flows and new addition links, Gao et al. (2005) transformed the bi-level programming of DNDP into a general nonlinear problem and thus traditional constrained optimization algorithms can be used. Solanki et al. (1998) decomposed the DNDP into a sequence of sub-problems and presented a quasi-optimization heuristic

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algorithm. Furthermore, heuristic/meta-heuristic approaches were studied to solve DNDP, including ant system/cooperating agents algorithm (Poorzahedy and Abulghasemi, 2005), genetic algorithms (Drezner and Wesolowsky, 2003; Kim and Kim, 2006) and so on. Some methods of hybrid meta-heuristic were also designed and compared among each other (Poorzahedy and Rouhani, 2007). More recently, global optimal algorithms for NDP have generated interest amongst researchers. Wang and Lo (2010) employed single-level mixed-integer linear programming (MILP) to approximating continuous network design problem (CNDP), which dealt with continuous expansion of existing links. The nonlinearity of travel time function was removed by applying a convex-combination based piecewise linear approximation. Luathep et al. (2011) further extended this method to solve mixed network design problem (MNDP), which is a combination of CNDP and DNDP. The DUE condition was depicted by a variational inequality (VI) problem and a cutting constraint based algorithm was proposed to seek the optimal solution. Farvaresh and Sepehri (2011) developed a single-level mixed-integer linear programming by transforming the lower-level DUE constraints into the equivalent Karush–Kuhn–Tucker (KKT) condition. Li et al. (2012) presented a global optimal approach for CNDP based on the concept of gap function and penalty. Wang et al. (2013) developed a NDP model with discrete multiple capacity levels to address the problem of adding an optimal number of lanes to existing candidate links. Furthermore, Fontaine and Minner (2014) proposed a solution method based on bender decomposition to solve linearized discrete network design problem. A global optimal method is designed by making use of the relationship between user equilibrium traffic assignment and system optimal principle. Szeto et al. (in press) address a sustainable road network design problem with land use transportation interaction over time. Liu and Wang (2015) proposed a global optimization solution approach for CNDP with stochastic user equilibrium travel flow pattern.

In previous studies, the discrete network design problems (DNDP) assume pre-determined road capacity for candidate link addition, while only addressing the issue that whether or not a new link will be constructed. However, it is more interesting to answer the question that whether or not a new link should be added, and simultaneously, if added, what is the optimal link capacity. In this paper, we exploit a DNDP problem with consideration of link capacity optimization, which aims to optimize the network performance via determining which links should be added from a set of candidate links and what capacities the new links to be constructed should have. The decision variables for a candidate link simultaneously include both discrete (binary) variables, which indicates whether the candidate link will be added or not, and continuous variables, i.e. the link capacity variables (the scenario with only discrete capacity levels is also considered in this paper). The DUE condition is used to describe the equilibrium traffic flow. Taking the advantage of variational inequality formulation in representing the DUE condition, this study firstly formulates a mathematical program with equilibrium constraints. Then, a global optimization method is proposed to solve the problem. As the transport network design problem is naturally formulated as an inherently nonlinear and non-convex problem, the advantage and benefit of finding the globally optimal solution is obvious, to ensure that the network design plan is exactly the "best plan" to achieve the targeted goal. Indeed, no previous studies have ever developed global optimization solution method for solving the transport network design problem presented in this paper, and this study could contribute in filling in this research gap in the literature. Noting that the nonlinearity of the problem stems from the bilinear terms and nonlinear travel time functions in the programming, this study applies two different techniques to deal with them. For the bilinear functions, we apply a Reformulation–linearization technique (Sherali and Adams, 1994, 1998) to transform them into a set of equivalent linear constraints; meanwhile, for the multi-variable travel time functions, we firstly take logarithm of them and then derive its mixed-integer linear relaxation through an outer-approximation technique. By doing so, a mixed-integer linear program (MILP) relaxation model is obtained, whose solution provides a tight lower bound of the original model solution. Then, a range reduction technique is applied to update and improve the lower bound until the gap between the lower bound and upper bound fulfills certain stopping criteria. The solution algorithm is proved to converge to the global optimal solution of the original problem.

This study considers a novel, yet more general NDP problem, which is sought to provide transportation network planners more indicative information not only on new candidate link additions, but also on optimal capacity of the new links, which are otherwise assumed to be given in previous DNDP studies. The developed model is more general formulation, which may include other conventional network design problems as particular cases. For example, when the capacity for each new link addition is given, this model will reduce to traditional DNDP in the literature; when the discrete variables on new link addition plan is predetermined, this problem is indeed a classical continuous network design problem (CNDP). Assuming road capacities to be continuous, the solutions of CNDP provide a "first-best" road capacity expansion plan. In practice, the CNDP modeling and solution algorithm is more useful when signalization or ramp metering is considered (Yang and Bell, 1998). Besides, in this study, it is also demonstrated that the model formulation can be used to solve the case of DNDP assuming discrete link capacity (discrete number of lanes) for new link additions. For the model formulation, which is still intrinsically nonlinear and noncovex, a global optimization algorithm is developed to solve the model to its exact global optimal solution. Specifically, the original model formulation is firstly relaxed into a mixed integer linear programing problem, whose solution provides the lower bound of the original problem. Then, the lower bound is updated and improved until the global optimization solution is obtained. In constructing the linear programming relaxation, reformulation and linearization technique and mixed-integer outer-approximation approach are adopted. In summary, this paper contributes to the literature in the following aspects: firstly, it provides a novel yet general network design problem formulation to address both the discrete link addition design and continuous road capacity design, which is not studied in previous researches (to our best knowledge). Secondly, a global optimization solution algorithm employing various linearization techniques is developed. Different from the global optimization algorithm used in previous studies (Wang and Lo, 2010 and Luathep et al., 2011), the solution method developed in this study is proved to be able to solve the real global optimum of the original problem,