Transportation Research Part E 70 (2014) 356–373

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

Robust hub network design problem

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ARTICLE INFO

Article history:
Received 20 June 2013
Received in revised form 1 July 2014
Accepted 6 August 2014

Keywords:
Hub network design
Robust optimization
Mixed integer conic quadratic programs

ABSTRACT

This paper presents a robust formulation for the uncapacitated single and multiple allocation hub location problem where demand is uncertain and its distribution is not fully specified. The proposed robust model is formulated as a mixed integer nonlinear program and then transformed into a mixed integer conic quadratic program. An efficient linear relaxation strategy is proposed which is found to deliver the optimal solutions for all the cases considered in this paper. Numerical experiments suggest location of more number of hubs when accounting for demand uncertainty using robust optimization compared to the deterministic setting.

1. Introduction

Hub and spoke network configurations are extensively utilized in logistics, freight, airline, and telecommunication infrastructures (Hernandez et al., 2011, Dobson and Lederer, 1993, Carello et al., 2004). To date a significant amount of research has been conducted studying different variants of the hub location problem – single allocation, multiple allocation, capacitated, uncapacitated – with various extensions (Alumur and Kara, 2008; Cambell and O'Kelly, 2012; Farahani et al., 2013).

Determining the location of hubs is a long term strategic freight planning problem which can be affected by uncertainties in demand. As has been well documented in the facility location, supply chain, and freight optimization literature, not accounting for uncertainty in inputs can lead to sub-optimal decisions (Snyder, 2006; Unnikrishnan et al., 2009). Stochastic optimization and robust optimization are the two main paradigms for dealing with uncertainty in mathematical programming models. Stochastic optimization relies on the distribution of the uncertain parameters while robust optimization is a distribution free approach aiming to find the worst case scenario with respect to a predefined uncertainty set (Ben-Tal and Nemirovski, 1999; El Ghaoui et al., 1998; Bertsimas et al., 2011).

This paper presents a robust optimization model for locating uncapacitated hubs in a network for both single and multiple allocation cases under uncertainty in demand. The robust optimization framework is adopted in this research as in several cases it is difficult to collect data to properly calibrate the probability distribution of demand. In this paper, the uncertain demand is assumed to lie in an ellipsoidal uncertainty set and is structurally characterized by a known mean and several independent variables (Chen et al., 2007, 2008). Each independent variable represents one source of uncertainty. The goal of the robust optimization strategy is to immunize the hub location decisions against the worst case demand uncertainty realization in the ellipsoidal uncertainty set. The mixed integer nonlinear program for the robust hub location problem is reformulated as a conic quadratic mixed integer program which can be solved efficiently using solvers such as CPLEX and

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http://dx.doi.org/10.1016/j.tre.2014.08.003
1366-5545/

MOSEK. An efficient linear relaxation is provided which is found to deliver the optimal solution for all the cases considered in this paper at reduced computational times.

The rest of the paper is organized as follows: Section 2 provides a brief review of hub location models as well as previous research addressing uncertainty in hub location problems. Section 3 introduces the deterministic uncapacitated single and multiple allocation formulation used in this research. Section 4 provides the robust modeling formulation and the linear relaxation for the robust counterpart. Section 5 describes the computational performance of the proposed formulations on Civil Aeronautics Board (CAB) dataset under different scenarios, and finally Section 6 summarizes the conclusions and findings and provides future research directions.

2. Literature review

O'Kelly (1986, 1987) proposed the first modeling formulation for cost minimizing hub network design. This initial quadratic integer formulation was then reformulated as an integer linear program considering both single and multiple allocation schemes by Campbell (1994, 1996). Subsequently Ernst and Krishnamoorthy (1996, 1998) proposed efficient variants of the hub location-allocation problem with fewer variables. The initial linear formulations of Campbell (1994, 1996) and Ernst and Krishnamoorthy (1998, 1996) were modified and extended in a number of directions by incorporating different features such as hub capacities (e.g. Ebery et al., 2000), hub as well as edge capacities (Sasaki and Fukushima, 2003), networking policies (Aykin, 1995), congestion at the hub (de Camargo and Miranda, 2012), etc. Recently, hub location formulations have been solved in the context of many practical research problems such as park and ride (Aros-Vera et al., 2013), carrier collaboration (Hernandez et al., 2011), rail networks (e.g. Jeong et al., 2007), multi-echelon supply chain networks (Shahabi et al., 2013) to only name a few. For comprehensive surveys on various hub location problems and its variants, the readers are referred to Alumur and Kara (2008), Cambell and O'Kelly (2012), and Farahani et al. (2013).

The significance of addressing parameter uncertainty has been intensively studied in the area of facility location network design problems (Snyder, 2006). However, the available literature dealing with uncertainty in the context of hub location problems is limited. Recently, Alumur et al. (2012) captured the effects of uncertainty in demand as well as the set-up cost by introducing a scenario based stochastic model and demonstrated the sub-optimality of the deterministic solution of hub location problems in the presence of uncertainty. Contreras et al. (2011) considered uncertain demand and transportation cost with known probability distribution for multiple allocation uncapacitated hub location problems. They solved the problem by Monte Carlo simulation framework combining sample average approximation (SAA) strategy with Benders decomposition algorithm. Their results confirmed that the SAA yields superior solutions when compared to the expected value equivalent problem and this difference is more significant at higher levels of variance for the uncertain transportation cost. Sim et al. (2009) considered stochastic travel time on each link and incorporated service level requirements through chance constraints in stochastic p-hub problems. Some papers studied stochastic hub location problems from the perspective of airline planning; Marianov and Serra (2003) modeled the hubs operation as an M/D/c queuing system and solved the proposed model by a heuristic method based on tabu search procedure. Yang (2009) incorporated seasonal variation of demand into air freight hub location problems and proposed a two stage stochastic program method as the solution methodology.

This paper proposes a robust modeling formulation for uncapacitated single and multiple allocation hub location problems with demand uncertainty. The demand distribution is unknown and only limited features of the demand distribution are available. One way to model uncertain parameters with limited information on underlying distribution function is by assuming that the uncertain parameter can be expressed as a linear combination of a known mean and several independent random variables (Chen et al., 2007). More specifically, Wagner et al. (2009) showed that such a linear affine combination for demand uncertainty is capable of capturing systematic and nonsystematic risk associated with freight demand. Consequently the uncertain demand is assumed to be affinely dependent on a known mean and a number of independent random variables. To the best of our knowledge, this is the first paper addressing robust strategy for hub location problems and considering uncertain demand with ambiguous distribution. The notation and the deterministic formulation for multiple and single allocation hub location problem used for constructing the robust counterpart is presented in the next section.

3. Notations and deterministic models

The robust formulation for the uncapacitated hub network design is developed on the deterministic model of Cánovas et al. (2006) and Hamacher et al. (2004) for multiple allocation and Skorin-Kapov et al. (1996) for single allocation case. The main advantage of the deterministic models of Cánovas et al. (2006), Hamacher et al. (2004) and Skorin-Kapov et al. (1996) over the other hub location formulations is that the demand uncertainty only affects the objective function in the robust formulation, and thus provides a more compact formulation for the robust counterpart. Moreover, the selected deterministic models are claimed to provide tight bounds for the relaxed linear problem. The formulations for the deterministic multiple allocation and single allocation hub location problems are provided next.