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Algebraic connectivity maximization of an air transportation network: The flight routes' addition/deletion problem

P. Wei [a,*], L. Chen [b], D. Sun [a]

[a] School of Aeronautics and Astronautics, Purdue University, West Lafayette, IN 47907, USA
[b] Department of Industrial Engineering, University of Louisville, Louisville, KY 40292, USA

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ABSTRACT

A common metric to measure the robustness of a network is its algebraic connectivity. This paper introduces the flight routes addition/deletion problem and compares three different methods to analyze and optimize the algebraic connectivity of the air transportation network. The Modified Greedy Perturbation algorithm (MGP) provides a local optimum in an efficient iterative manner. The Weighted Tabu Search (WTS) is developed for the flight routes addition/deletion problem to offer a better optimal solution with longer computation time. The relaxed semidefinite programming (SDP) is used to set a performance upper bound and then three rounding techniques are applied to obtain feasible solutions. The simulation results show the trade-off among the Modified Greedy Perturbation, Weighted Tabu Search and relaxed SDP, with which we can decide the appropriate algorithm to adopt for maximizing the algebraic connectivity of the air transportation networks of different sizes. Finally a real air transportation network of Virgin America is investigated.

1. Introduction

An air transportation network consists of the nodes that represent airports and the edges that represent the flight routes which directly link two airports (Wei and Sun, 2011; Guimera and Amaral, 2004; Vargo et al., 2010). It can be described as a graph G with n nodes and m edges. With the fact that if a direct flight route exists between airport v_i and airport v_j, normally the direct return flight route from v_j to v_i also exists (ICAO, 2010), G is constructed as an undirected graph, where the airports are indexed as $\{v_i | i = 1, 2, \ldots, n\}$ and the edge between airports v_i and v_j is named as e_{ij}.

In an air transportation network either a node failure or an edge failure may happen due to weather disturbance, long Ground Delay Program (GDP), long Airspace Flow Program (AFP), aircraft mechanical problem, upline flight delay/cancel and other unforeseen events. How to build a robust or well connected network, which has the ability to transport passengers between any two airports via one edge or through multiple edges under the unpredictable node or edge failures, is a practical problem that has significant economic impact. In this paper we measure and optimize the robustness of an air transportation network by computing its *algebraic connectivity*, which is one of the network metrics from graph theory research. Compared to the betweenness, degree and clustering coefficient that are defined on each node (Bigdeli et al., 2009), the algebraic connectivity is selected as the network robustness metric in this work because researchers have shown that it has the tightest bound to the network robustness in terms of node and edge connectivities and it is the most computational efficient network robustness metric (Jamakovic and Uhlig, 2007; Jamakovic and Mieghem, 2008; Byrne et al., 2009).

* Corresponding author.
E-mail addresses: weip@purdue.edu (P. Wei), lijian.chen@louisville.edu (L. Chen), dsun@purdue.edu (D. Sun).

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Traditionally, the *node connectivity* and the *edge connectivity* are the two metrics to evaluate a graph's robustness (Gibbons, 1985). The node (edge) connectivity of a graph G is the minimum number of node (edge) deletions sufficient to disconnect G.

In order to show the limitation of node (edge) connectivity metric, two different topologies are shown, where Fig. 1a is an N-node line topology and Fig. 1b is an N-node star topology. The node connectivities for both topology formations are 1 and so are the edge connectivities. However, the star topology should be more robust than the line topology because in Fig. 1b the network will be disconnected only when the central node fails, while in Fig. 1a any node failure can cause the network to disconnect except the two end nodes. The robustness features of the two topologies are intuitively different. But neither the node connectivity nor the edge connectivity can observe the difference between these two topologies.

The algebraic connectivity is defined by Fiedler as the second smallest Laplacian eigenvalue of a graph (Fiedler, 1973). According to this definition, when $N = 4$, the algebraic connectivities of Fig. 1a and b are 0.586 and 1 respectively, which show that the star topology is more robust than the line topology. This example demonstrates that the algebraic connectivity is a finer measurement for network robustness. Researchers from graph theory and network theory have also proved that the algebraic connectivity provides better resolution on how well a graph or network is connected and it is a fair measurement of the network robustness (Jamakovic and Uhlig, 2007; Byrne et al., 2009).

The air traffic demand is expected to continue its rapid growth in the future. The Federal Aviation Administration (FAA) estimated that the number of passengers is projected to increase by an average of 3% every year until 2025 (FAA, 2010). The expanding traffic demand on the current air transportation networks of different airlines will cause more and more flight cancelations with the limited airport resources and airspace capacities. As a result, more robust air transportation networks are desired to sustain the increasing traffic demand for each airline and for the entire National Airspace System (NAS). That is the major motivation of this work.

In reality imposing weights on edges is necessary because the weights bring more information to an air transportation network. Usually different routes have different edge (link) strength. For example, the route failure rate between JFK and BOS during summer is higher than that between SFO and LAX because of the crowded northeastern airspace (AFP is more frequent) and more summer thunder storms. Another example is that a shorter route is easier to fail than a longer transcontinental route because: (a) airlines usually put larger aircraft on transcontinental route and these aircraft are more robust to weather disturbance; (b) airlines are more likely to cancel shorter route flights because the flight frequency on a shorter route is higher therefore the passengers on the canceled flight are easier to be reaccommodated to later flights. In summary, the routes have different possibilities to fail and in this study we use different edge weights to describe the varying link strength. A stronger edge under random failure is assigned with a greater edge weight value while smaller edge weights are assigned to those edges that are easier to fail.

The goal of this work is to maximize the algebraic connectivity in a weighted air transportation network under given constraints. Although the maximized algebraic connectivity value is abstract, the optimized air transportation network design is applicable. The methods developed in this work are expected to be implemented to measure and to enhance the robustness of air transportation networks. With these methods the decision makers from airline companies can maintain or modify the structure of an existing regional or nationwide network and design strategies for the future development of their air transportation network.

The rest of the paper is organized as follows. The related work from literature is presented in Section 2. In Section 3 we formulate the flight routes addition/deletion problem and show that the weighted problem is NP-hard. In Section 4 the heuristic algorithm for the unweighted graph is extended to solve the weighted problem. The tabu search algorithm is developed for the flight routes addition problem in Section 5. In Section 6 the relaxed semidefinite programming (SDP) method is introduced and three different rounding techniques are discussed. In Section 7 we evaluate the performances of our algorithms via numerical simulations. A real air transportation network of Virgin America is investigated in Section 8. Section 9 concludes this paper.

2. Related work

The air transportation network and its robustness have been studied over the last several years. Guimera and Amaral (2004) first studied the scale-free graphical model of the air transportation network. Conway (2004) showed that it was better to describe the national air transportation system or the commercial air carrier transportation network as a system-of-systems. Bonnefoy (2008) showed that the air transportation network was scale-free with aggregating multiple airport

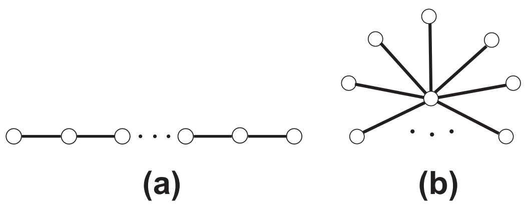

Fig. 1. *N*-node line topology and star topology.