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A novel hybrid-link-based container routing model

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ABSTRACT

Container routing determines how to transport containers from their origins to their destinations in a liner shipping network. Container routing needs to be solved a number of times as a subproblem in tactical-level decision planning of liner shipping operations. Container routing is similar to the multi-commodity flow problem. This research proposes a novel hybrid-link-based model that nests the existing origin-link-based and destination-link-based models as special cases. Moreover, the hybrid-link-based model is at least as compact as the origin-to-destination-link-based, origin-link-based and destination-link-based models in the literature.

1. Introduction

Liner shipping companies deploy containerships on regularly scheduled services to transport containers. Containerships in liner shipping have to sail according to the planned schedule no matter whether they are fully loaded or not. Liner shipping services are usually weekly, which means that each port of call is visited on the same day of every week. Once the weekly liner shipping services are designed, they are operated for a period of three to six months. Therefore, it is important for liner shipping companies to design efficient services as a large proportion of the total operating cost is fixed once the services are designed.

Liner shipping decision problems can be classified as strategic, tactical, and operational (Christiansen et al., 2004, 2013; Meng et al., 2013). Fleet size and mix (e.g., Meng and Wang, 2011), alliance strategy (e.g., Agarwal and Ergun, 2010) and network design (e.g., Fagerholt, 1999, 2004; Shintani et al., 2007; Imai et al., 2009; Gelareh et al., 2010; Gelareh and Pisinger, 2011; Reinhardt and Pisinger, 2012; Plum et al., 2013) are strategic-level decision problems. Network alteration (e.g., Wang and Meng, 2013), fleet deployment (e.g., Meng and Wang, 2012; Wang and Meng, 2012a), schedule design (e.g., Qi and Song, 2012), and speed optimization (e.g., Psaraftis and Kontovas, 2013) are tactical decision problems. Operational decisions include problems such as container booking/routing (e.g., Song and Dong, 2013) and ship rescheduling (e.g., Yan et al., 2009; Brouer et al., 2013b).

Container routing occurs at both the operational level and the tactical level. Container routing determines how to transport containers from their origins to their destinations in a liner shipping network (Wang et al., 2013b). Take Fig. 1 as an example. It shows a liner shipping network consisting of three ship routes with fixed port rotations. Containers from Singapore to Hong Kong can be transported either on ship route 1 or ship route 2. If there are many containers to be transported from Singapore to Jakarta, then containers from Singapore to Hong Kong should be transported on ship route 2 to reserve the capacity on ship route 1 for containers from Singapore to Jakarta. In addition to different ship routes on which containers can be transported from origin to destination, another complicating factor is transshipment. For instance, containers from Hong

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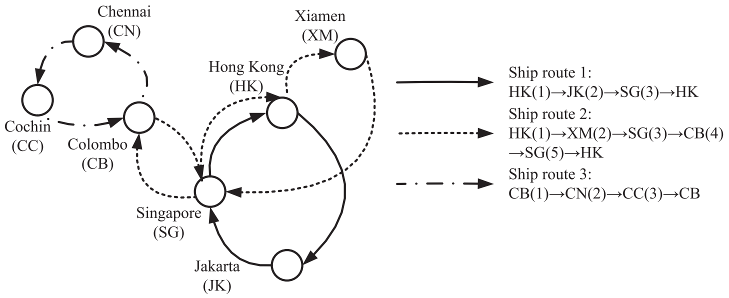

Fig. 1. An illustrative liner shipping network. *Source*: Wang, 2013.

Kong to Colombo can be transported on ship route 2, or they can be transported on ship route 1 to Singapore and transshipped to ship route 2 and then transported to Colombo. The choice of direct shipment on ship route 2 is preferable because the latter involves a high transshipment cost at Singapore. However, if there are many containers to be transported from Hong Kong to Xiamen or from Xiamen to Singapore, then the choice of transshipment at Singapore from ship route 1 to ship route 2 has to be adopted. Consequently, it is not an easy task to determine the optimal container routing.

Container routing is not only significant to liner shipping companies as an independent problem at the operational level, more often than not, it serves as a subproblem in a number of tactical-level decision problems such as network alteration and fleet deployment. In the tactical-level decision problems, container routing must be solved iteratively and hence the computational efficiency of container routing models is vital. Consequently, it is important to develop models for container routing that are compact and can be easily solved.

Container routing problems are very similar to multi-commodity flow problems (MCF) studied in the field of operations research (e.g., Tomlin, 1966; Ahuja et al., 1993; Gamst et al., 2010). MCF can be solved in polynomial time. However, there are often extra conditions that have to be satisfied, making the problem NP-hard. Moreover, the sizes of many practical applications are extremely large. Therefore, a number of specialized algorithms have been developed, most of which use a decomposition strategy that is based on duality and relaxation of coupling constraints. The main motivations for decomposition are (i) to reduce the problem to smaller sub-problems and (ii) to parallelize or distribute computations. We refer to Ouorou et al. (2000) for an overview of solution algorithms on MCF.

The container routing problem and MCF are not identical. For instance, the MCF with an upper-bounded path length is NP-hard (Gamst et al., 2010). However, the container routing problem with an upper-bounded path travel time is still polynomially solvable. This is because unlike MCF, the liner shipping services have a weekly frequency.

There are generally two types of container routing models (or MCF models): path-based models (Brouer et al., 2011; Song and Dong, 2012; Wang and Meng, 2012b; Wang et al., 2013a) and link-based models. Path-based models need to enumerate all possible paths or generate dynamically the profitable paths for containers to be transported from origin to destination. By contrast, the number of variables in link-based models increases polynomially with the size of the liner shipping network. The advantage of path-based models is that side constraints can be easily handled.

It should be noted that in network design for MCF, it is usually to determine whether a link should be added or not. That is, the network design under MCF is to determine whether the capacity of a link is 0 (no construction cost) or a fixed positive number (with a fixed construction cost). Hence, both link-based models and path-based models are used in network design under MCF. In liner shipping network design, a link cannot be added separately, because a shipping service is a loop, where the links are connected, have the same capacity, and provide a weekly frequency. As a result, path-based model may be difficult to handle in liner shipping network design. Hence, to date we are unaware of any studies on liner shipping network design that uses path-based formulations for container routing. Recently, Plum et al. (2013) made a breakthrough in liner shipping network design by proposing an exact solution method for the most general problem settings. Their container routing model is somewhere between link-based and path-based formulations.

The objective of this research is to develop a novel and compact link-based model, which we call hybrid-link-based model. It can be more efficiently solved by general-purpose commercial solvers than other link-based models and can be applied to many situations.

The remainder of the paper is organized as follows: Section 2 reviews existing link-based models in the literature. Section 3 proposes a novel hybrid-link-based model that requires fewer variables than existing models. Section 4 develops a linear programming model to obtain the optimal choice of origins and destinations for the hybrid-link-based model. Section 5 reports numerical experiments. Section 6 concludes.

2. Existing link-based models

Before presenting existing link-based models, we describe the container routing problem and define relevant parameters.

Consider a set $\mathcal{R}$ of ship routes, regularly serving a group of ports denoted by the set $\mathcal{P}$. Ship route $r \in \mathcal{R}$ can be expressed as: