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MODELLING OIL ABSORPTION DURING POST-FRYING COOLING
II: Solution of the Mathematical Model, Model Testing and Simulations

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The mathematical models that describe the immersion-frying period and the post-frying cooling period of an infinite slab or an infinite cylinder were solved and tested. Results were successfully compared with those found in the literature or obtained experimentally, and were discussed in terms of the hypotheses and simplifications made. The models were used as the basis of a sensitivity analysis. Simulations showed that a decrease in slab thickness and core heat capacity resulted in faster crust development. On the other hand, an increase in oil temperature and boiling heat transfer coefficient between the oil and the surface of the food accelerated crust formation. The model for oil absorption during cooling was analysed using the tested post-frying cooling equation to determine the moment in which a positive pressure driving force, allowing oil suction within the pore, originated. It was found that as crust layer thickness, pore radius and ambient temperature decreased so did the time needed to start the absorption. On the other hand, as the effective convective heat transfer coefficient between the air and the surface of the slab increased the required cooling time decreased. In addition, it was found that the time needed to allow oil absorption during cooling was extremely sensitive to pore radius, indicating the importance of an accurate pore size determination in future studies.

Keywords: frying; model solution; model testing; oil uptake; post-frying cooling.

INTRODUCTION

The oil content of fried foods is an important quality parameter: high values are incompatible with recent consumer trends toward healthier food and low-fat products (Bouchon and Pyle, 2004). To obtain products low in fat, it is essential to understand the mechanisms involved during the process, so that oil migration into the structure can be effectively minimized. Recent research has shown that, even though the counter-flows of water vapour and oil are related to each other, they are not synchronized (Bouchon *et al.*, 2003), since oil is mostly absorbed within the crust during the cooling period (Ufheil and Escher, 1996; Moreira *et al.*, 1997; Aguilera and Gloria-Hernández, 2000; Bouchon *et al.*, 2003). In fact, it is suggested that the processes of water loss by boiling and oil uptake occur sequentially: significant oil uptake can only occur once the steam blanket around the frying food has collapsed.

In the companion paper (Bouchon and Pyle, 2005), a predictive mechanistic model was developed with the aim of predicting oil absorption kinetics during post-frying cooling. The process of water removal and porous crust development were described by a transient moving-front model, followed by cooling. Oil uptake was considered to be a pressure driven flow mediated by capillary forces, which was only allowed when a positive pressure gradient had developed. The model was developed for two geometries: an infinite slab and an infinite cylinder, and can be used to study oil absorption in any product that satisfies the assumptions made.

In this paper, the mathematical models developed in part one of this paper (hereafter referred to as Bouchon and Pyle, 2005) are solved and tested to ascertain their applicability. To do so, results are compared with those found in the literature (Farkas *et al.*, 1996a) or obtained experimentally, and are discussed in terms of the hypotheses and simplifications made. None of the coefficients in the model were adjusted to fit the experimental results: all the parameters were either taken from the literature or measured/inferred from separate experiments. Finally, a sensitivity analysis

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was carried out to examine the effects of physical properties and process variables on temperature distribution, crust development (moisture loss) and oil penetration driving force.

SOLUTION OF THE MATHEMATICAL MODEL

The models presented in part one of this paper are formulated as a mixed system of integral, partial differential (i.e., distributed parameter) and algebraic equations (IPDAEs). The mathematical models were programmed and solved using gPROMS (version 1.8.2 for Windows 98, Process Systems Enterprise Ltd, London, UK), a software package designed for modelling and simulation of processes with both discrete and continuous characteristics, which permits the solution of complex mathematical models, which are distributed and described over rectangular domains in terms of mixed IPDAEs (Oh and Pantelides, 1996). The systems of IPDAEs are solved numerically using the method of lines (MOL), which involves the discretization of the distributed equations with respect to all spatial domains, thus resulting in mixed sets of time-dependent ordinary differential and algebraic equations. The modelling language allows one to define the type of spatial approximation method to be used to solve the system (*viz.* finite differences or finite elements), as well as the granularity and the order of the approximation. Since none of the models involved significant second derivative terms, a finite difference discretization method was used throughout, using a second order central finite difference approximation over a uniform grid of 10 μm intervals. This ensured convergence of the numerical solution and no differences were detected when further reducing the mesh. The time integration step length in gPROMS was determined automatically by the numerical method and was actually varied at each integration step to ensure that the specified accuracy was satisfied (Oh and Pantelides, 1996).

Model Solution for Immersion-Frying Period

As explained in Bouchon and Pyle (2005), the model for the immersion-frying period is associated with the period of time that begins when the product is submerged into the oil and lasts until it leaves the bath. During that process three main stages occur: (1) an initial sensible heating period, (2) an initial water evaporation period, and (3) the evaporation period itself. These phases were modelled individually and consecutively. The initial sensible heating period (unsteady heat conduction with constant effective thermal diffusivity) was easily programmed and the numerical solution was compared to the analytical solution, which is well known for both geometries (Incropera and DeWitt, 1996). The initial water evaporation period corresponds to the transition period during which the regime changes from natural convection to boiling heat transfer and is defined by a simple algebraic equation, which assumes that the convective heat is entirely used to evaporate the water at the outer surface until a thin layer is formed. It was found that an initial layer thickness of 1 μm or smaller did not affect the results and consequently this value was used. As a result, the temperature profile obtained in the initial sensible heating period was considered to be the initial temperature distribution within the core, whereas the small initial crust region (1 μm thickness) was assumed to be at the evaporation temperature (100°C).

Once the initial crust is developed, the evaporation front starts to propagate towards the interior of the product. The resultant mathematical model comprises two partial differential equations describing the flux of heat in each of the two regions (Bouchon and Pyle, 2005), the crust and the core, separated by the moving front. This is a Stefan problem, which has the mathematical difficulty of involving a moving boundary. To overcome this inconvenience, the approach taken by Farkas *et al.* (1996b), which consists of fixing the advancing evaporation front by means of a Landau coordinate transformation, was adopted. Accordingly, the two partial differential equations and the respective boundary and initial conditions, were defined in terms of new ζ and η domains, for the core and crust regions, respectively, to replace the former x domain, using the following transformations:

$$0 \le \zeta = \frac{x}{\delta(t)} \le 1 \qquad \text{for } 0 \le x \le \delta(t) \tag{7}$$

$$0 \le \eta = \frac{x - \delta(t)}{D - \delta(t)} \le 1 \qquad \text{for } \delta(t) \le x \le D \tag{8}$$

where the coordinate x represents the axial or the radial distance, $\delta(t)$ represents the distance of the evaporation front and D represents either half of the thickness of the infinite slab ($L/2$) or the radius of the infinite cylinder (R), depending on the geometry under study.

As a result, the distribution domains are redefined in dimensionless form and are forced to lie between 0 and 1, and consequently, the moving front is now defined in terms of a 'fixed' position, leading to the following set of equations for an infinite slab:

$$\frac{\partial T_{\text{core}}}{\partial t} - \frac{\zeta}{\delta}\frac{\partial T_{\text{core}}}{\partial \zeta}\frac{\partial \delta}{\partial t} = \frac{\alpha_{\text{core}}}{\delta^2}\frac{\partial^2 T_{\text{core}}}{\partial \zeta^2} \tag{9}$$

$$\frac{\partial T_{\text{curst}}}{\partial t} + \frac{\eta - 1}{L/2 - \delta}\frac{\partial T_{\text{curst}}}{\partial \eta}\frac{\partial \delta}{\partial t} = \frac{\alpha_{\text{crust}}}{(L/2 - \delta)^2}\frac{\partial^2 T_{\text{crust}}}{\partial \eta^2} \tag{10}$$

and the following set of equations for an infinite cylinder:

$$\frac{\zeta\delta^2}{\alpha_{\text{core}}}\left\{\frac{\partial T_{\text{core}}}{\partial t} - \frac{\zeta}{\delta}\frac{\partial T_{\text{core}}}{\partial \zeta}\frac{\partial \delta}{\partial t}\right\} = \frac{\partial T_{\text{core}}}{\partial \zeta} + \zeta\frac{\partial^2 T_{\text{core}}}{\partial \zeta^2} \tag{11}$$

$$\frac{\eta(R-\delta)+\delta}{\alpha_{\text{crust}}}\left\{\frac{\partial T_{\text{crust}}}{\partial t} + \frac{\eta - 1}{R - \delta}\frac{\partial T_{\text{crust}}}{\partial \eta}\frac{\partial \delta}{\partial t}\right\} = \frac{1}{R-\delta}\frac{\partial T_{\text{crust}}}{\partial \eta} + \frac{\eta(R-\delta)+\delta}{(R-\delta)^2}\frac{\partial^2 T_{\text{crust}}}{\partial \eta^2} \tag{12}$$

Both sets are associated with the following set of boundary conditions:

$$\left.\frac{\partial T_{core}}{\partial \zeta}\right|_{\zeta=0} = 0 \quad \forall t > 0 \tag{13}$$

$$h_{\text{f}}(T_{\text{oil}} - T_{\text{crust}}|_{\eta=1}) = \frac{k_{\text{crust}}}{(D-\delta)}\left.\frac{\partial T_{\text{crust}}}{\partial \eta}\right|_{\eta=1} \quad \forall t > 0 \tag{14}$$

$$T_{\text{core}}|_{\zeta=1} = T_{\text{crust}}|_{\eta=0} = T_{\text{evap}} \quad \forall t > 0 \tag{15}$$

$$\frac{k_{\text{crust}}}{(D-\delta)}\left.\frac{\partial T_{\text{crust}}}{\partial \eta}\right|_{\eta=0} = \frac{k_{\text{core}}}{\delta}\left.\frac{\partial T_{\text{core}}}{\partial \zeta}\right|_{\zeta=1} - \rho_{\text{w}}\lambda\varepsilon\frac{d\delta}{dt} \quad \forall t > 0 \tag{16}$$