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A note on "Berth allocation considering fuel consumption and vessel emissions"

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ABSTRACT

Du et al. [Du, Y., Chen, Q., Quan, X., Long, L., Fung, R.Y.K., 2011. Berth allocation considering fuel consumption and vessel emissions. Transportation Research Part E 47, 1021–1037] dealt with a berth allocation problem incorporating ship' fuel consumption minimization. To address the difficulty posed by the power function between fuel consumption rate and sailing speed, they formulated a tractable mixed-integer second-order cone programming model. We propose two quadratic outer approximation approaches that can handle general fuel consumption rate functions more efficiently. In the static quadratic outer approximation approach, the approximation lines are generated a priori. In the dynamic quadratic outer approximation approach, the approximation lines are generated dynamically. Numerical experiments demonstrate the advantages of the two approaches.

1. Introduction

Du et al. (2011) proposed a continuous berth allocation problem (BAP) with ships' fuel consumption minimization by taking into account two objectives: one aims to minimize the fuel consumption of all ships; the other one seeks to minimize the total departure delay of all ships. To obtain the efficient frontier, the first objective is minimized subject to the constraint that the second objective is not worse than certain values, and the whole model is solved as a single objective optimization model.

To keep this note relatively self-contained, we give a short description of the problem addressed by Du et al. (2011). Given a set of ships denoted by V in the berth plan, the fuel consumption rate for ship $i \in V$ at the speed s can be estimated by

$$c_i^0 + c_i^1 s^{\mu_i} \tag{1}$$

where c_i^0, c_i^1 and μ_i are nonnegative parameters [see Eq. (8) in Du et al. (2011)]. The distance of ship $i \in V$ to the berth is denoted by m_i (nautical miles). The arrival time a_i for ship $i \in V$ is bounded by the interval $[\underline{a}_i, \bar{a}_i]$, which is determined by the sailing distance m_i and the ship's maximal and minimal sailing speeds. To minimize fuel consumption, the arrival time a_i (or, equivalently, the speed) for each ship $i \in V$ is optimized. The objective that aims to minimize the fuel consumption of all the ships can be formulated as

$$\min_{a_i} f_1 = \sum_{i\in V}\left[c_i^0 + c_i^1\left(\frac{m_i}{a_i}\right)^{\mu_i}\right]a_i = \sum_{i\in V}(c_i^0 a_i + c_i^1 m_i^{\mu_i} a_i^{1-\mu_i}) \tag{2}$$

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subject to the necessary linear constraints. Hence, the BAP with fuel consumption minimization is formulated as a mixed-integer nonlinear programming model (the integer variables are associated with berth allocation).

1.1. Second-order cone programming approach

A major challenge in solving the above model is the nonlinear objective function f_1 shown in Eq. (2) resulting from the power function (1) between fuel consumption rate and sailing speed. To take advantage of state-of-art mixed-integer second-order cone programming (MISOCP) solvers such as CPLEX, Du et al. (2011) successfully reformulated the nonlinear fuel consumption objective f_1 as a linear objective subject to a group of second-order cone programming (SOCP) constraints and obtained a computationally tractable model. A second-order cone program is a convex optimization problem with linear objective function and convex quadratic constraints. Linear program is a special case of the second-order cone program (see, e.g., Alizadeh and Goldfarb, 2003, for an introduction to SOCP). They first reformulated f_1 as:

$$\min_{a_i} f_1 = \sum_{i\in V}(c_i^0 a_i + c_i^1 m_i^{\mu_i} q_i) \tag{3}$$

subject to

$$q_i \geq a_i^{1-\mu_i}, \quad i \in V \tag{4}$$

They chose the fuel consumption rate parameter $\mu_i \in \{3.5, 4.0, 4.5\}$, and transformed the constraints shown in Eq. (4) to the SOCP constraints. For example, when $\mu_i = 4.5$, after introducing new decision variables and several computational steps [see Eqs. (23)–(27) in Du et al. (2011)], Eq. (4) can be transformed to

$$\begin{aligned} &\|(2\mu_{i1}, a_i - 1)\|_2 \leq a_i + 1, \|(2\mu_{i2}, \mu_{i1} - q_i)\|_2 \leq \mu_{i1} + q_i, \|(2\mu_{i3}, a_i - \mu_{i2})\|_2 \leq a_i + \mu_{i2} \\ &\|(2\mu_{i4}, \mu_{i1} - 1)\|_2 \leq \mu_{i1} + 1, \|(2, \mu_{i3} - \mu_{i4})\|_2 \leq \mu_{i3} + \mu_{i4}, a_i, q_i \geq 0, \mu_{i1}, \mu_{i2}, \mu_{i3}, \mu_{i4} \geq 0, \quad i \in V \end{aligned} \tag{5}$$

As a result, the mixed-integer nonlinear BAP with fuel consumption optimization model is transformed to an MISOCP model, which has both modeling accuracy and computational advantages.

1.2. Objectives and contributions

The objective of this study is to propose an alternative methodology to overcome this computational difficulty posed by the nonlinear relation between sailing speed and fuel consumption rate. In particular, we propose two quadratic outer approximation approaches that can handle general fuel consumption rate functions more efficiently. In the static quadratic outer approximation approach, the approximation lines are generated a priori. In the dynamic quadratic outer approximation approach, the approximation lines are generated dynamically. Numerical experiments demonstrate the advantages of the two approaches.

2. A quadratic outer approximation approach

In view of the special structure of Eq. (4), we can use a group of SOCP constraints to approximate it. First, we derive a quadratic outer approximation function to approximate the function $\tilde{q}_i(a_i) := a_i^{1-\mu_i}$ in the neighborhood of a particular point $\hat{a}_i \in [\underline{a}_i, \bar{a}_i)$ for ship $i \in V$, as shown in Fig. 3 with $\underline{a}_i, \bar{a}_i = [20, 40]$, $\mu_i = 4.237$ and $\hat{a}_i = 30$ (note that we assume $\hat{a}_i \neq \bar{a}_i$ for reasons that will become obvious later). The quadratic function, denoted by $Q_i(\hat{a}_i, a_i)$, passes the point $(\hat{a}_i, \tilde{q}_i(\hat{a}_i))$. Hence, it can be represented by

$$Q_i(\hat{a}_i, a_i) = k_i(\hat{a}_i)(a_i - \hat{a}_i)^2 + l_i(\hat{a}_i)(a_i - \hat{a}_i) + \hat{a}_i^{1-\mu_i} \tag{6}$$

To determine the coefficients $k_i(\hat{a}_i)$ and $l_i(\hat{a}_i)$, we require that $Q_i(\hat{a}_i, a_i)$ has the same slope as $\tilde{q}_i(a_i)$ at the point $\hat{a}_i$. Thus, we obtain

$$l_i(\hat{a}_i) = (1 - \mu_i)\hat{a}_i^{-\mu_i} \tag{7}$$

Finally, we seek for $k_i(\hat{a}_i)$ such that the gap between the function $\tilde{q}_i(a_i)$ and $Q_i(\hat{a}_i, a_i)$ is minimized. Observing that (i) $\tilde{q}_i(a_i)$ and $Q_i(\hat{a}_i, a_i)$ take the same value and also have the same first-order derivative at $\hat{a}_i$, (ii) the second-order derivative of $\tilde{q}_i(a_i)$, $\mu_i(\mu_i - 1)a_i^{-\mu_i-1}$, is positive and decreasing, and (iii) the second-order derivative of $Q_i(\hat{a}_i, a_i)$ is constant, we have two lemmas and two propositions described below. Lemma 1 and Lemma 2 provide an intuitive explanation of Proposition 1. Proposition 2 is a theoretical extension of Proposition 1. We omit the proofs of these lemmas and propositions because they are straightforward.

Lemma 1. *If the second-order derivative of* $Q_i\hat{a}_i, a_i$*, i.e.,* $2k_i(\hat{a}_i)$*, equals the second-order derivative of* $\tilde{q}_i(a_i)$ *at* $\hat{a}_i$*, i.e.* $\mu_i(\mu_i - 1)\hat{a}_i^{-\mu_i-1}$*, then* $\tilde{q}_i(a_i) > Q_i(\hat{a}_i, a_i)$ *for any* $\underline{a}_i \in [_i, \hat{a}_i)$ *and* $\tilde{q}_i(a_i) < Q_i(\hat{a}_i, a_i)$ *for any* $a_i \in (\hat{a}_i, \bar{a}_i]$*.*

The function $Q_i(\hat{a}_i, a_i)$ in Lemma 1 with $[\underline{a}_i, \bar{a}_i] = [20, 40]$, $\mu_i = 4.237$ and $\hat{a}_i = 30$ is shown in Fig. 1.