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A Lagrangian heuristic for the real-time vehicle rescheduling problem

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ABSTRACT

When a public transit vehicle breaks down on a scheduled trip, one or more vehicles need to be rescheduled to serve that trip and other service trips originally scheduled for the disabled vehicle. In this paper, the *vehicle rescheduling problem* (VRSP) is investiaged to consider operating costs, schedule disruption costs, and trip cancellation costs. The VRSP is proven to be NP-hard, and a Lagrangian relaxation based insertion heuristic is developed. Extensive computational experiments on randomly generated problems are reported. The results show that the Lagrangian heuristic performs very well for solving the VRSP.

1. Introduction

Public transportation systems are susceptible to unexpected costs and delays due to unforeseen events, such as vehicle breakdowns and traffic accidents. While minor vehicle failures can be repaired quickly, serious failures require long repair times and sometimes even result in towing the disabled vehicle for lengthy repairs. Vehicle breakdowns in public transportation systems require that the passengers from the disabled vehicle and those expected on the remaining part of the trip be picked up. In addition, the original schedule of the breakdown vehicle may be deteriorated to the extent where the fleet plan may need to be adjusted in real-time depending on the current state of what is certainly a dynamic system. The implementation of new technologies (e.g., automatic vehicle locaters, the global positioning system, geographical information systems, and wireless communication) in public transit systems makes it possible to provide real-time information and implement real-time vehicle rescheduling algorithms at low cost.

In some previous studies (Li et al., 2004, 2007a,b), the authors have introduced the vehicle rescheduling problem and proposed some solution approaches. The problem was partially modeled as a sequence of static vehicle scheduling problems and was pseudo-polynomially solved by using an auction algorithm (Li et al., 2004, 2007b). A decision support system was developed to facilitate a practical application for rescheduling trucks for solid waste collection (Li et al., 2007a). Although the auction-based algorithm performs well for a large number of trips and vehicles, the solution approach is based on two assumptions: (i) scheduled trips, except the disrupted trip, cannot suffer delays; and (ii) there are no restrictions on the number of trips that may be reassigned. However, these assumptions may be restrictive in some real-world situations. For example, the vehicle breakdown may delay more than one trip when the trips that the breakdown vehicle is scheduled to cover are at a considerable distance from the depot and other vehicles, or when there is no extra vehicle at the depot. In

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addition, if only operating and delay costs are minimized to obtain a new schedule, the initial schedule might be considerably altered. In many situations, it is crucial to keep the changes in schedule low, because, for example, large changes may make crew scheduling difficult if familiarity with itineraries is important. This paper expands the previous models of VRSP by simultaneously considering schedule disruptions and delays of multiple trips, so that the methods can be used in more real-world applications.

Dynamic vehicle routing and scheduling problems have gained increasing attention since the late eighties. Tabu search (Ichoua et al., 2000), genetic algorithm (Haghani and Jung, 2005), assignment and insertion-based heuristics (Shieh and May, 1998; Fleischmann et al., 2004), approximate dynamic programming (Spivey and Powell, 2004), dynamic column generation (Chen and Xu, 2006) and nearest-vehicle heuristic (Du et al., 2005) have been proposed for the dynamic vehicle routing problems. Huisman et al. (2004) propose a dynamic vehicle scheduling approach to avoid starting trips late, in a public transportation system with recurrent traffic jams. However, the focus of dynamic routing and scheduling problems is mainly on online requests and uncertain travel times, while the unexpected events, such as vehicle breakdowns, are not the major concern.

Disruptions management has also been receiving substantial attentions in the airline schedule recovery. Some typecial studies include Carlson (2000), Jarrah et al. (1993), Lettovský (1997), Rosenberger et al. (2003), Teodorović and Stojković (1995), Yan and Yang (1996) and Yu et al. (2003). Although there are interesting and useful ideas on the development of disruption management in the airline community, the problems experienced by airlines are significantly more difficult, due to complicated operational and safety requirements involved. The computational requirements of the developed method are as consequence very large. It may be unrealistic to use such approaches directly for a simpler vehicle rescheduling problem.

The major contributions in this paper are: (i) the arc-based formulation of the single depot VRSP and the NP-hardness proof; and (ii) the development of a Lagrangian heuristic for solving the VRSP, incorporating Lagrangian relaxation, a subgradient search and an insertion-based primal heuristic.

This paper is organized as follows. The formal description of the problem and mathematical formulation are given in Section 2. The Lagrangian-based heuristic is described in Section 3. Section 4 presents computational experiments to compare the performance of the developed algorithm with an intuitive approach. Summary of the results and areas of future research are discussed in Section 5.

2. Problem statement and formulation

For completeness, we summarize some definitions and notation introduced in Li et al. (2007a,b). A vehicle in a transit system (especially bus) performs a service trip after service trip in its schedule, where a *service trip* is defined by a specified starting time and location where the vehicle begins its service of picking up and dropping off passengers at a sequence of bus stops and an ending time and location when the bus either *dead heads* to the depot or goes to the starting point of another service trip. A *deadheading* trip is a movement of an empty vehicle to a destination without picking up or dropping off any passenger. The current trip of the disabled vehicle will be referred to as the *cut trip*. The place and time the vehicle breaks down on the cut trip will be referred to as the *breakdown point* and *breakdown time*, respectively. The vehicle that serves the remaining passengers in the cut trip will be referred to as the *backup vehicle*. If the backup vehicle is currently servicing a trip or has just completed a trip, then this trip may be referred to as the *backup trip*. A *path* is a sequence of service trips in which all consecutive pairs of trips in the sequence are *compatible* in the sense that there is sufficient time available to finish the preceding service trip and then deadhead to the consecutive service trip. Such a path defines a schedule of a vehicle. In general, any two trips i and j are a *compatible pair of trips* if the same vehicle can reach the starting point and time of trip j after it finishes trip i. We will use a binary variable β to indicate whether a service trip is compatible with another service trip. If $\beta(i,j) = 1$, trips i and j are compatible. A *cut path* is the path on which the disabled vehicle is originally scheduled.

There are two possible breakdown situations in the vehicle rescheduling problem: (i) the cut trip is a service trip, or (ii) a deadheading trip. In the first case, the solution is to send a backup vehicle to the breakdown point and to complete the cut trip from the breakdown point. However, since it is possible that some trips have common itineraries, the vehicles that cover compatible itineraries after the breakdown point may serve the passengers incidentally. Therefore, the following situation needs to be avoided in rescheduling: a backup vehicle changes its original schedule and travels towards the breakdown point but, by the time it reaches there, all the passengers from the disabled vehicle have been incidentally picked up by vehicles that cover compatible itineraries with the remaining part of the cut trip. In case of (ii), when the cut trip is a deadheading trip, the solution is to assign a backup vehicle for the starting location of the next trip of the deadheading vehicle. The determination of backup trip candidates is based on the available capacity of the involved vehicles, the time to reach the breakdown point and the compatibility of itineraries among trips.

In the static vehicle scheduling problem (VSP), there is no need to consider assigning a specific vehicle to trips, since all vehicles are identical, and we can assign them arbitrarily after the schedule is determined. Unlike the VSP, the VRSP has to take into account this issue, since operating vehicles are out of the depot and at different locations when a vehicle becomes disabled. Corresponding operating costs, which include the costs to reach the breakdown point, are different. Each vehicle in the midst of operations can be viewed as a *pseudo-depot* with one available vehicle, and the vehicle can be dispatched to any future trip from its current location if it is a deadheading trip or from the ending point of its current trip if it is a service trip. We consider here only the following operational strategy: a vehicle currently on a service trip can only change its schedule