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Unit root test against ESTAR with deterministic components

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ABSTRACT

This article intends to notify that asymptotic distributions of the nonlinear unit root test statistics must be rigorously treated if deterministic components are included in the estimated regression. The simple inductive argument of replacing the standard Brownian motion by either the demeaned or demeaned and detrended ones usually adopted in the literature is invalid. New results on the asymptotic distributions of the t-ratio to test the null of the unit root against the nonlinear exponential smooth transition autoregressive (ESTAR) with deterministic components are provided.

1. Introduction

Dissatisfaction with the linear ARMA framework, within which unit root tests are applied, has recently prompted econometricians to consider nonlinear alternatives to develop unit root tests. Kapetanios, Shin, and Snell (2003) popularized an ADF type statistic to test the null of unit root against the stationary exponential smooth transition autoregressive (ESTAR) alternative, which has been found in international monetary economics as an effective way to describe real exchange rates dynamics. In contrast, Rothe and Sibbertsen (2006) developed Phillips-Perron type unit root tests against the ESTAR alternative, in which error terms are allowed to be strong mixing.

Both Kapetanios et al. (2003) and Rothe and Sibbertsen (2006) began with the leading case without deterministic components to derive asymptotic distributions of their test statistics. However, they claim that in the presence of deterministic components, through a bypass remark mimicking the arguments used in linear unit root tests, the asymptotic distribution follows only by replacing the standard Brownian motion with either the demeaned motion or the demeaned and detrended motion. We argue in this article that this is not the case, since what actually should be demeaned or detrended is the cubic of lag series. Asymptotic distributions of the nonlinear unit root test statistics must be rigorously considered if deterministic components are included in the estimated regression. New results on the asymptotic distribution of the t-ratio proposed in Kapetanios et al. (2003) and Rothe and Sibbertsen (2006) are provided when the estimated regression includes deterministic components.

The rest of this article is organized as follows. Section 2 describes the basic ESTAR model, assumptions, and the test statistics of the ESTAR unit root test. In section 3, new results on the asymptotic distributions with deterministic components included in the estimated regression are proven. Section 4 then concludes.

It should be noted that in this paper, $\|X\|_p = (E(|X|^p))^{\frac{1}{p}}$ denotes the L_p norm, "$\xrightarrow{p}$" ("$\xrightarrow{a.s.}$") denotes convergence in probability (almost surely), W(r) refers to a standard Brownian motion defined on $r \in [0,1]$, "$\Rightarrow$" and represents weak convergence in distribution.

2. The model, assumption and test statistics

Consider a univariate ESTAR of order 1 model, ESTAR(1), which is described in Kapetanios et al. (2003) and Rothe and Sibbertsen (2006):

$$y_t = d_{kt}^{'}\psi + \rho y_{t-1} + \gamma y_{t-1} G(y_{t-1};\theta) + \varepsilon_t, \quad (1)$$

$$G(y_{t-1};\theta) = 1 - \exp\left(-\theta y_{t-1}^2\right), \quad (2)$$

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where $\mathbf{d}_{kt}$ is the deterministic component, ρ and γ are unknown scalar parameters, and is a vector parameter conformable with $\mathbf{d}_{kt}$. We distinguish three kinds of deterministic components, $\mathbf{d}_{1t} = \{\varnothing\}, \mathbf{d}_{2t} = (1), \mathbf{d}_{3t} = (1, t)'$. The transition function $G(y_{t-1};\theta\ )$ is a scaling parameter which controls the shift of regimes and is assumed to be of exponential form. The parameter $\theta \geq 0$ determines the speed of transition between regimes. Kapetanios et al. (2003) and Rothe and Sibbertsen (2006) both impose $\rho = 1$ and$0<\gamma<1$, which means that the two regimes of the ESTAR model are assumed to correspond to a unit root process and a stable AR(1), respectively. Following Rothe and Sibbertsen (2006), the disturbance term "t" is allowed to follow various forms of temporal dependence and also heteroskedasticity.

Assumption 1 For some$p > \beta > 2$, $\{\varepsilon_t\}$ is a zero mean, strong mixing sequence with mixing coefficients α_m of size $-p\beta/(p-\beta)$ and$\sup_{i\geq 1}\|\varepsilon_i\|_p = C < \infty$. In addition, $T^{-1}E((\sum_1^T \varepsilon_t)^2) \to \lambda^2 > 0$ *as* $T \to \infty$.

Remark 1 Assumption 1 assures that the functional central limit theorem (FCLT) and results regarding the convergence to stochastic integrals will apply to certain normalized partial sums of $\{\varepsilon_t\}$. The parameter λ^2 is the long-run variance. Although the second moment of $\{\varepsilon_t\}$ is not assumed to be constant over time, McLeish (1975) strong law of large asserts that:

$$\frac{1}{T}\sum_{t=1}^{T}\varepsilon_t^2 \xrightarrow{a.s.} \sigma^2 \equiv \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}E\left(\varepsilon_t^2\right). \tag{3}$$

where the parameter σ^2 is interpreted as the average error variance. Note that $\sigma^2 = \lambda^2$ when $\{\varepsilon_t\}$ is a sequence of i.i.d. variables.

The null we are interested in is when the process y_t follows a random walk. Under the ESTAR framework (1), Kapetanios et al. (2003) and Rothe and Sibbertsen (2006) proposed testing the null H0: $\theta = 0$ against the alternative H1: $\theta > 0$. Since the parameter γ is only identified under the alternative, the first order Taylor series approximation of the exponential function around zero is applied to overcome this nuisance parameter problem. Combining this approximation with our restrictions on ρ and γ, leads to the auxiliary regression:

$$\varDelta y_t = \mathbf{d}_{kt}'\psi + \phi y_{t-1}^3 + \mu_t \tag{4}$$

where $\phi = \gamma\theta$ and $u_t = \varepsilon_t + \gamma y_{t-1}R(y_{t-1},\theta)$, with $R(y_{t-1},\theta)$ being a remainder term from the Taylor approximation. It is evident that $u_t = \varepsilon_t$ under the null. In Equation (4) our hypotheses are equivalent to:

$$H_0 : \phi = 0 \ \ vs.\ \ H_1 : \phi < 0. \tag{5}$$

By the Frisch-Waugh-Lovell Theorem (Davidson & MacKinnon, 2004, Theorem 2.1), estimating ϕ from the auxiliary regression (4) is equivalent to the estimate from the following regression:

$$\mathbf{M}_k\varDelta\mathbf{y} = \mathbf{M}_k\mathbf{y}_{-1}^3\phi + \mathbf{u}, \tag{6}$$

where k = 1, 2, 3 denotes the cases with deterministic components d_{1t}, d_{2t}, d_{3t}, respectively. We use the notations $\varDelta\mathbf{y} = (\varDelta y_1, \varDelta y_2, \cdots, \varDelta y_T)'$,$\mathbf{y}_{-1}^3 = (y_0^3, y_1^3, \cdots, y_{T-1}^3)'$, $\mathbf{u} = (u_1, u_2, \cdots u_T)'$, and the annihilators $\mathbf{M}_1 = \mathbf{I}_T, \mathbf{M}_2 = \mathbf{I}_T - \mathbf{1}_T(\mathbf{1}'uT\mathbf{1}T)^{-1}\mathbf{1}^{uT}$, with $\mathbf{1}_T = (1, \cdots, 1)\prime$, and $\mathbf{M}_3 = \mathbf{I}_T - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$, with $\mathbf{X} = (\mathbf{1}^{uT}, (1, \cdots, T)\prime)$. The *t*-ratio to test the null is based on the OLS estimate of ϕ in the auxiliary regression (6):

$$t_{\widehat{\phi}k} = \frac{\widehat{\phi}_k}{\sqrt{\widehat{\sigma}_k^2\mathbf{y}_{-1}^3\mathbf{M}_k\mathbf{y}_{-1}^3}}, \tag{7}$$

where:

$$\widehat{\phi}_k = \frac{\mathbf{y}_{-1}^3\mathbf{M}_k\varDelta\mathbf{y}}{\mathbf{y}_{-1}^3\mathbf{M}_k\mathbf{y}_{-1}^3}, \tag{8}$$

$$\widehat{\sigma}_k^2 = \frac{1}{T}\left(\varDelta\mathbf{y} - \widehat{\phi}_k\mathbf{y}_{-1}^3\right)'\mathbf{M}_k\left(\varDelta\mathbf{y} - \widehat{\phi}_k\mathbf{y}_{-1}^3\right). \tag{9}$$

From Equation (6), it is obvious that the demeaned or detrended series of y_{t-1}^3 and$\varDelta y_t$ are used to estimate ϕ, rather than those of y_{t-1} and $\varDelta y_t$. Consequently, the bypass arguments made in Kapetanios et al. (2003) and Rothe and Sibbertsen (2006) to derive the asymptotic distribution in cases with deterministic components are incorrect. A rigorous treatment to the asymptotic distribution of nonlinear unit root tests in which the estimated regression includes deterministic components is needed. In general, if the estimated regression is of the following form with $g(\bullet)$ a nonlinear function:

$$y_t = d_{kt}'\psi + g(y_{t-1}) + error,$$

a direct bypass argument to induct the asymptotic distribution is not adequate, since $\mathbf{M}g(y_{t-1}) \neq g(\mathbf{M}y_{t-1})$, where $\mathbf{M}$ is a suitable annihilator.

3. Asymptotic results with deterministic components

If ε_t satisfies Assumption 1, a version of the FCLT guarantees the normalized partial sum process of ε_t to converge in distribution to a Brownian motion (White, 2001, Theorem 7.18):

$$Y_t(r) \equiv \frac{1}{\sqrt{T}}\sum_{i=1}^{[T_r]}\varepsilon_i = \frac{1}{\sqrt{T}}y[T_r] \Rightarrow \lambda W(r). \tag{10}$$

Provided that Assumption 1 holds with $\beta = 6$, Rothe and Sibbertsen (2006) showed that $t_{\widehat{\phi}_1}$ has the following asymptotic distribution under the null[1]:

$$t_{\widehat{\phi}_1} \Rightarrow \frac{\frac{1}{4}\lambda^4 W(1)^4 - \frac{3}{2}\lambda^2\sigma^2\int_0^1 W(r)^2 dr}{\sqrt{\lambda^6\sigma^2\int_0^1 W(r)^6 dr}} \tag{11}$$

In what follows, we will show that the asymptotic distributions of $t_{\widehat{\phi}_2}$ and $t_{\widehat{\phi}_3}$ cannot be simply obtained by replacing the standard Brownian motion with the demeaned Brownian motion, $W^u(r) = W(r) - \int_0^1 W(s)ds$, or the demeaned and detrended Brownian motion, $W^T(r) = W(r) - \int_0^1 (4-6s)W(s)ds + r\int_0^1(-6+12s)W(s)ds$.

We now establish some convergence results that are essential for further asymptotic development.

Lemma 1 Let Assumption 1 hold with $\beta = 6$ and the null H0 being true, then as$T \to \infty$, we have:

[1] When the error terms $\{\varepsilon_t\}$ are an i.i.d. sequence, we have$\sigma^2 = \lambda^2$ and thus Kapetanios et al.'s (2003) result follows.