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Article

Pricing Asian power options under jump-fraction process

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ABSTRACT

A framework for pricing Asian power options is developed when the underlying asset follows a jump-fraction process. The partial differential equation (PDE) in the fractional environment with jump is constructed for such option using general Itô's lemma and self-financing dynamic strategy. With the boundary condition, an analytic formula for the option with geometric average starting at any time before maturity is derived by solving the PDE, and the option with arithmetic average is evaluated in Monte Carlo simulation using control variate technique with the help of the above analytic solution. Overwhelming numerical evidence indicates that the technique proposed is computationally efficient and dramatically improves the accuracy of the simulated price. Moreover, this study will pave a novel way to copy with the option contracts based on thinly-traded assets like oil, or currencies or interest rates.

Tasar las opciones energéticas de Asia por el método de fracciones discontinuas

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RESUMEN

Se desarrolla un marco para tasar las opciones energéticas asiáticas sometiendo el valor del activo subyacente a un método de fracciones discontinuas. La ecuación en derivadas parciales (EDP) en el entorno fraccional con salto se construye para una opción dada utilizando la fórmula general de Itô y una estrategia dinámica de autofinanciación. Con la condición límite, resolviendo la EDP se deriva una fórmula analítica para la opción con una media geométrica que empieza en cualquier momento antes de la madurez, y la opción con media aritmética se evalúa con simulación de Monte Carlo utilizando variables de control apoyadas en la citada solución analítica. Hay abrumadora evidencia numérica de que la técnica propuesta es eficiente en tiempo de cálculo y mejora espectacularmente la precisión del precio simulado. Es más, este estudio abrirá un nuevo camino que seguir en los contratos de opción de compra basados en bienes tan poco negociables como el petróleo, las divisas o los tipos de interés.

1. Introduction

All kinds of exotic options arise one after another in the environment of volatile financial market. Asian power options are successful. They have become widely used in the fields of stock, commodity, energy and foreign exchange. Kemna and Vorst (1990) proposed an analytic expression for Asian options with geometric average using the partial differential equation (PDE) approach, on this basis, geometric average as control variable employed in the Monte Carlo simulation method (Boyle, 1977) was used to obtain satisfactory result for pricing Asian options with arithmetic average. Chen and Lyuu (2007) came up with a close-form solution for arithmetic Asian option using the approximation of arithmetic average through geometric average appeared. The approximate approach works as well as the Monte Carlo simulation approach

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but accuracy remains problematic for high volatility and/or long maturity cases. In addition, lattice binomial method (Hull & White, 1993; Neave & Turnbull, 1993) has been proposed to handle Asian options. But it has a dramatic computational cost because a large number of possible realizations of the payoff must be considered. Among the above most common methods to price Asian options, Monte Carlo method is rather simple to implement and can provide standard errors for the estimates to measure quality, and it further achieves a satisfactory level of accuracy with the enhancement of control variate technique for more complex arithmetic average option (see Boyle et al, 1997). At the same time, the analytic solution of the Asian option with geometric averaging is indispensable in the control variate technique and PDE approach (Alziary et al., 1997; Zhang, 2001) provides an accurate result for geometric average option without computationally expensive when the PDE to be solved has three or four independent variables. As far as power option is concerned, Blenman and Clark (2005) explicitly solve for the price of the power option to exchange one asset for another under the equivalent martingale measure in which they specified that the price of power call is equal to the price of the power exchange option when the power of another asset is zero.

Most of the academic researches on such exotic options assume that the underlying asset evolves as a continuous diffusion process. This implies that logarithmic returns of the asset are normal random variables. However, empirical evidences in Jorion (1988), Bates (1996), Pan (2002), Chernov et al. (2003), and Eraker (2004) indicate the presence of discontinuous jump in asset price when significant new information or catastrophic events arise. The jump-diffusion process is widely used to model jumps of the price movement and was introduced to option evaluation by Merton (1976) and Gukhal (2004). In recent years, many empirical studies on capital market also show that the logarithmic returns on financial assets are not normally distributed but the distribution with excess kurtosis and fat tail. Moreover, price series on financial assets are not stochastic motion but long-range dependence. Peters (1989) found the fractal structure and non recurrent phenomenon in both stock and exchange rate market and proposed the hypothesis of fractional market. Fractional Brownian motion, as a family of Gaussian processes, can give a satisfactory description of the price dynamics of the underlying asset because it has two important properties of self-similarity and long-range dependence. Considering fractional Brownian motion is neither a Markov process nor a semi martingale, Duncan et al. (2000) built up the fractional-Itô-integral to analyze it. Furthermore, Hu (2003) proofed that the option market under the fractional Brownian motion is perfect without arbitrage opportunity using the Wick integration and gave European option pricing formula at arbitrary time. Indeed, some authors have used the fractional Brownian motion to capture the behavior of underlying asset and to obtain fractional Black-Scholes formulae for pricing options including Necula (2002), Bayraktar et al. (2004) and Meng and Wang (2010).

To better describe the evolution of asset price, the combination of Poisson jumps and fractional Wiener process is introduced in this paper. The jump fractional process is consistent with an efficient market where major information arrives infrequently and randomly. In addition, this process is capable of capturing the empirically observed distributions of asset price changes that are leptokurtic, skewed, long memory and have fatter tails than comparable normal distributions, and provides a good explanation for volatility smile effect of log normally based Black-Scholes model. That is, the implied volatility varies with moneyness and maturity.

The problem of pricing option when the underlying asset value is driven by a jump fractional process was solved by Xiao et al. (2010) who derived a pricing model for currency option value. Things are more complicated in the case of exotic path-dependent option such as Asian power options developed in this paper whose payoff depends on the geometric or arithmetic average of the underlying asset raised to power. Such option represents a simultaneous generalization of Asian options as well as power option both of which play an important role in the risk management and incentive contract (see Zhang, 1997, Tompkins, 1999). The average feature embedded in power option makes Asian power option less subject to price manipulation thus hedging nonlinear risk arising in option positions from changing level of implied volatility and smoothing randomness inherent in the stock price so that the managers can be evaluated more fundamentally in the incentive contract for indexed executive option compensation. Despite many literatures on Asian option, there is litter work on Asian power option.

The objective of this paper is to study the pricing of Asian power options with geometric and arithmetic. Meanwhile we capture the behavior of the underlying asset using the jump-fraction process and follow the control variate technique whose chief advantage is its high accuracy and efficiency. The outline of the rest of the paper is as follows: The next section derives the analytical formula for the Asian power options with geometric average using PDE approach after giving the assumption of pricing environment. Section III demonstrates how the analytical solution as control variable is implemented in the Monte Carlo simulation to obtain an accurately simulated price of the Asian power option with arithmetic average. Conclusions are presented in the final section.

2. The valuation model

Consider a complex and flexible financial economy where information arrives both continuously and discontinuously. This is modeled as a continuous component with the features of "asymmetric leptokurtic" and "long memory" and as a discontinuous component with abnormal fluctuation in the price process. Assume that the asset pay dividends, the price process can hence be specified as a superposition of these two components and can be represented as;

$$dS_t = \left(\mu_t - q_t\right) S_t + \sigma S_t (dB_t^H + dN_t) \tag{1}$$

where μ_t and q_t are time-dependent parameter respectively denoting expected yield rate and dividend rate. σ is volatility; $B_t{}^H$ is a fractional Brownian motion with Hurst parameter $H \in (0, 1)$ which is Centered Gaussian process with mean zero and covariance $\mathrm{cov}\left(B_t^H B_s^H\right) = \left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right)$; Q_t is a Poisson process with intensity λ, dependent of $B_t{}^H$, Nt is Poisson compensation process and equals $Q_t - \lambda t$.

Theorem 1 Set $W_t = B_t{}^H + N_t$, $f(t,\phi) \in C^{1,2}$ $(R_+ \times R \to R)$ and $f(t,W_t)$, $\int_0^t \frac{\partial f}{\partial \tau}(\tau, W_\tau) d\tau$, $\int_0^t \frac{\partial^2 f}{\partial \phi^2}(\tau, W_\tau) d\tau$, and $\int_0^t \frac{\partial^2 f}{\partial \phi^2}(\tau, W_\tau) \tau^{2H-1} d\tau$ belonging to $L^2(P)$, then

$$f(t,W_t) = f(0,0) + \int_0^t \left[\frac{\partial f}{\partial \tau}(\tau, W_\tau) + \left(H\tau^{2H-1} + 0.5\lambda\right)\frac{\partial^2 f}{\partial \phi^2}(\tau, W_\tau)\right] d\tau + \int_0^t \frac{\partial f}{\partial \phi}(\tau, W_\tau) dW_\tau \tag{2}$$

Proof: See Appendix A

Theorem 2: The solution of the stochastic differential equation (1) equals

$$S_t = S_0 \exp\left[\int_0^t \left(\mu_\tau - q_\tau\right) d\tau - \sigma^2 t^{2H}/2 - \lambda\sigma^2 t/2 + \sigma W_t\right] \tag{3}$$

Proof: Let

$f\left(t, W_t\right) = S_0 \exp\left[\int_0^t \left(\mu_\tau - q_\tau\right) d\tau - \sigma^2 t^{2H}/2 - \lambda\sigma^2 t/2 + \sigma W_t\right]$, then $dS_t = df(t, W_t)$, the theorem can be proven from Theorem 1.